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Paper

Evolutionary Shape Optimization of Flexbeam Sections of a Bearingless Helicopter Rotor

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ABSTRACT: The shape optimization of composite flexbeam sections of a bearingless helicopter rotor is studied using a finite element (FE) sectional analysis integrated with an efficient evolutionary optimization algorithm called particle swarm assisted genetic algorithm (PSGA). The sectional optimization framework is developed by automating the processes for geometry and mesh generation, and the sectional analysis to compute the elastic and inertial properties. Several section shapes are explored, modeled using quadratic B-splines with control points as design variables, through a multiobjective design optimization aiming minimum torsional stiffness, lag bending stiffness, and sectional mass while maximizing the critical strength ratio. The constraints are imposed on the mass, stiffnesses, and critical strength ratio corresponding to multiple design load cases. The optimal results reveal a simpler and better feasible section with double-H shape compared to the triple-H shape of the baseline where reductions of 9.46%, 67.44% and 30% each are reported in torsional stiffness, lag bending stiffness, and sectional mass, respectively, with critical strength ratio greater than 1.5.

Key Words: Flexbeam, Cross-section, Bearingless rotor, Shape optimization, Evolutionary algorithm

1. INTRODUCTION

The advances in composites technology have paved the way for the tailoring of the constitutive properties of helicopter rotor blades which impact the blade aeroelastic behavior. The coupled interaction among various disciplines, such as aerodynamics, dynamics, aeroelasticity and structures, poses a great challenge for the design of rotor blades. The bearingless main rotor (BMR) hub system lacks the conventional mechanical hinges and the pitch bearing which are used to control the blade dynamic behavior, resulting in advantages such as lower part count, lower weight, and less maintenance cost [1,2]. The BMR hub system (Fig. 1) consist of two main components: flexbeam (Fig. 2) and torque tube. The former has a torsionally soft open section which allows the flap and lag bending, and torsional motions while the latter has a torsionally stiff closed thin-walled section to control the blade pitch motion.

The flexbeam being a crucial component needs to be designed with low mass, low torsional stiffness, low bending stiffnesses and high strength to allow large twist motion with low dynamic stresses [2]. The flexbeam is made up of composite materials as the properties can be easily tailored by changing the geometric shape and fiber angle layup. The cross-sectional optimization, such as those of wind turbine/helicopter rotor blades, in general involves a large number of design variables along with many constraints. The design variables normally include the sectional shape as well as the layup sequence of composite materials.

The open section shapes with complex composite layup often face manufacturing difficulties. There are two major approaches which have been used for the optimization of beam cross-sections: the traditional approach of sizing and composite stacking sequence optimization, and a modern topology optimization approach. In the former, the design shape is fixed by choosing from a set of candidate shapes, and sizing and material layup optimization is performed using either low-cost gradient-based local optimizers or more expensive gradient-free global optimizers such as genetic algo-

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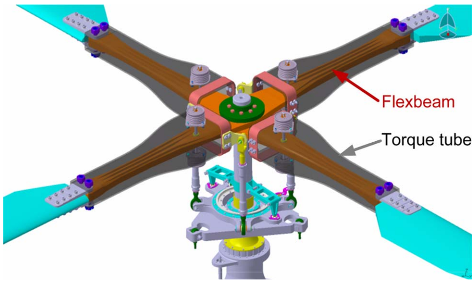

Fig. 1. Bearingless main rotor hub system

Fig. 2. Baseline flexbeam

rithm (GA). Since the design shape is fixed, better optimal shapes cannot be explored through this methodology. This approach has been followed by Ganguli and Chopra [3], Kathiravan and Ganguli [4], Li *et al.* [5], and Blasques and Stolpe [6]. The latter approach involves the discretization of initial domain into a fixed number of finite elements, and the material layout as well as the composite ply layup is optimized for the each element. Although innovative optimal shapes can be achieved using this approach, it requires extensive computation with the number of design variables equal to the number of discretized finite elements. The works related to topology optimization of beam sections have been reported by Kim and Kim [7], and Blasques and Stolpe [8]. In addition to the aspects mentioned previously, another component which drives the efficiency of the search is the optimization algorithm. Most of the previous works either used a gradient-based local optimizer such as SQP (sequential quadratic programming) or a simpler heuristic global optimization algorithm such as GA. These algorithms may not guarantee the global optimum solution due to certain limitations: (a) GA has a slow convergence rate to reach the optimal solution and may converge to artificial optimum; and (b) gradient-based methods such as SQP are unsuitable for problems with discrete design variables, multiple local optima, and discontinuous functions. The hybrid optimization algorithm PSGA [9] is used in the present study that intends to overcome these issues and search for the global optimum efficiently.

In the present study, the shape optimization of composite homogeneous flexbeam sections of a bearingless helicopter rotor is attempted by utilizing a finite element sectional analysis Ksec2d [10] and a gradient-free optimization algorithm PSGA [9]. The various shapes are investigated including H-shape, double-H shape and triple-H shape sections, which are represented by varying the number of control points of quadratic B-splines. A unidirectional layup of composite material is considered. A multiobjective design formulation is explored combined into a single objective function guided by the flexbeam design requirements. The constraints are imposed on the stiffnesses, sectional mass, and critical strength ratio for multiple design load cases. The manufacturing constraint is incorporated by considering the design variables as discrete or integer multiples.

2. SECTION OPTIMIZATION FRAMEWORK

Fig. 3 shows the overall flowchart of the section optimization framework. The framework is developed by the integration of various analysis tools in Fortran 90. MSC. Patran [11] is used to model the geometry and FE mesh of the cross-section. The modeling procedure is automated using Patran Command Language (PCL) [11] and the mesh is optimized using Cuthill-Mckee method available in MSC. Patran. The sectional stiffness and mass properties are computed using finite element analysis code Ksec2d [10]. The output of the cross-sectional analysis is generated in a text format while the geometry, mesh, and the sectional distributions of torsional out-of-plane warping displacement, strains and stresses are generated in a format suitable for the visualization in ParaView [12].

The procedure for the flexbeam section optimization using the PSGA is described as follows. First, the material and the fiber orientation database along with the sectional load cases are initialized. In addition, a cross-section template file written in PCL is constructed and provided as an input. The input file contains the variables related to the geometry, FE mesh, and material properties. PSGA generates the candidate solutions in the specified design search space. The cross-sectional geometry and mesh data are subsequently created for the design solutions using MSC. Patran which is run in batch mode (without graphical user interface), and the data are exported in

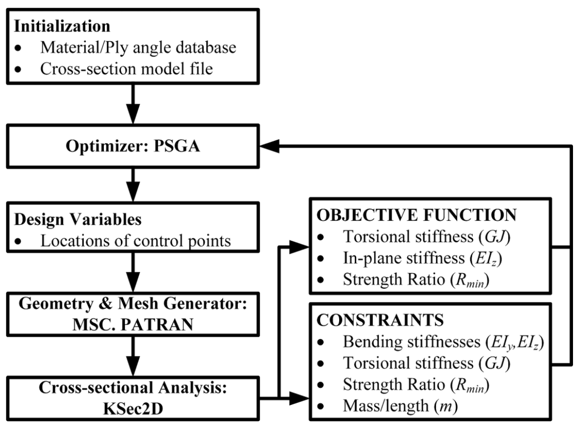

Fig. 3. Section optimization framework

a 'neutral' file format [11]. In this step, the discretization of the section is carried out using a mixed mesh consisting of both 6-node triangular and 8-node quadrilateral elements. The sectional input data are then fed into the sectional analysis code Ksec2d to compute the sectional mass and stiffness properties, and the strength ratio. The resulting sectional outputs are used to evaluate the objective function and constraint violation. The updated design solutions are then ranked according to the feasibility represented by the constraint violation and the corresponding objective function values. The optimization loop is continued until the optimum solution is reached.

2.1 Cross-sectional analysis

The FE cross-sectional analysis Ksec2d [10] is applicable for nonhomogeneous isotropic as well as quasi-isotropic composite beams. The formulation is based on the elasticity theory considering the Saint Venant's beam assumptions. The approach includes the effects of classical elastic couplings and non-classical out-of-plane torsional warping. The effects of transverse shear deformation and in-plane warping are neglected. The stiffness matrix is computed at Euler-Bernoulli level for bending and Vlasov level for torsion. The analysis is also capable of calculating the sectional mass per unit length and inertial properties. The recovery of sectional strains and stresses can be performed for given loading conditions. The failure analysis of both isotropic and composite materials is incorporated to compute the strength ratio based on von-Mises criterion for isotropic materials, and Tsai-Wu criterion for composite materials.

For the beams with no elastic couplings, the cross-sectional stiffness matrix $\mathbf{K}$ is given by

$$\mathbf{K} = \begin{bmatrix} EA & 0 & 0 & 0 & 0 \\ 0 & GJ & 0 & 0 & 0 \\ 0 & 0 & EI_y & 0 & 0 \\ 0 & 0 & 0 & EI_z & 0 \\ 0 & 0 & 0 & 0 & EC_\omega \end{bmatrix} \tag{1}$$

where EA is the extensional stiffness, GJ is the torsional stiffness, EI_y and EI_z are the flap and lag bending stiffnesses, respectively, and EC_ω is the warping stiffness. For the complete details of the sectional analysis, the reader is referred to [10].

2.2 Particle swarm assisted genetic algorithm (PSGA)

PSGA [9] is a gradient-free algorithm having the following salient features: (a) It can handle constraints effectively using a feasible population-based relaxation scheme; (b) It is applicable to problems with continuous/discrete and real/integer design variables; (c) It is more efficient compared to other state-of-the-art algorithms in searching for the global optimum solution due to the rank-based multi-parent crossover; (d) The stagnation check helps in avoiding the local optimum solutions and assists in reaching the global optimum; and (e) The parameters are self-adaptive requiring no tuning based on the type and size of the problems.

A general constrained optimization problem can be defined in mathematical form as

Minimize:

$$f(\mathbf{x}) \tag{2}$$

subject to:

$$g_j(\mathbf{x}) \le 0, \ \ j = 1, \ldots, m \tag{3}$$

$$h_k(\mathbf{x}) = 0, \ \ k = 1, \ldots, p \tag{4}$$

with bounds for design variables as:

$$x_i^L \le x_i \le x_i^U, \ \ i = 1, \ldots, n \tag{5}$$

where $f(\mathbf{x})$ is the objective function, $g_j(\mathbf{x})$ are the inequality constraints, $h_k(\mathbf{x})$ are the equality constraints, and x_i are the design variables with x_i^L and x_i^U as lower and upper bounds, respectively.

The constraint violation $\Phi(\mathbf{x})$ is defined as

$$\Phi(\mathbf{x}) = \sum_{j=1}^{m} \max[0, g_j(\mathbf{x})] + \sum_{k=1}^{p} |h_k(\mathbf{x})| \tag{6}$$

For the constrained optimization, there are several ways to handle the given linear or nonlinear constraints such as penalty function-based approach or feasibility-based approach. In PSGA, the constraints are initially relaxed such that the solutions are considered feasible if constraint violation is less than a certain value. For details, the reader may refer to [9].

2.3 Problem formulation

The optimization problem is formulated according to the design requirements as given by [1,2]:

- Low torsional stiffness (GJ)
- Low lag bending stiffness (EI_z)
- High critical strength ratio (R_{cr})
- Low mass (m)

For the multiobjective design of flexbeam section, the torsional stiffness, lag bending stiffness, critical strength ratio, and sectional mass are considered in the objective function. The constraints are imposed in the form of upper limits on the bending stiffnesses, torsional stiffness, and sectional mass, and lower limit on the critical strength ratio. Since the extensional stiffness is generally very high to allow any extensional motion, no direct constraint is imposed on its value. However, it must be mentioned that the strength ratio implicitly takes into account the stresses due to extensional loading which may affect the optimal shape of the section. The maximization problem of critical strength ratio is transformed into a minimization problem by inserting a negative sign with the objective function. The multiobjective optimization problem is constructed into a single objective problem by introducing weight factors as

Minimize:

$$f(\mathbf{x}) = w_1 \frac{GJ(\mathbf{x})}{GJ_0} + w_2 \frac{EI_z(\mathbf{x})}{EI_{z0}} + w_3 \frac{m(\mathbf{x})}{m_0} - w_4 \frac{R_{cr}(\mathbf{x})}{1.5} \tag{7}$$

subject to:

$$\frac{EI_y(\mathbf{x})}{EI_{y0}} \le 1,\ \frac{EI_z(\mathbf{x})}{EI_{z0}} \le 1,\ \frac{GJ(\mathbf{x})}{GJ_0} \le 1$$

$$\frac{R_{\min}(\mathbf{x})}{1.5} \ge 1,\ \frac{m(\mathbf{x})}{m_0} \le 1 \tag{8}$$

with bounds for design variables as:

$$\mathbf{x}^L \le \mathbf{x} \le \mathbf{x}^U \tag{9}$$

where w_1, w_2, w_3, w_4 are the weight factors, $\mathbf{x}$ is the design variables vector with $\mathbf{x}^L$ and $\mathbf{x}^U$ as the lower and upper bounds, respectively, R_{cr} is the minimum strength ratio, and subscript '0' indicates baseline properties.

The geometric dimensions of the section constitute the design variables for the structural shape optimization. It is to be noted that all the design variables are considered as discrete in the form of integer multiples of a base value which represent the manufacturing constraints.

3. RESULTS AND DISCUSSION

Numerical experiments are conducted by employing the developed section optimization framework to examine various optimal shapes. The critical strength ratio is computed using Tsai-Wu failure criterion for unidirectional composite materials. To this end, the multiple design load cases are considered corresponding to the hover and forward flight conditions of the bearingless rotor. For the present optimization, the weight factors for the objective function are $w_1 = 0.4$, $w_2 = 0.2$, $w_3 = 0.1$, and $w_4 = 0.3$, where a higher value signifies higher importance of the corresponding objective. The sectional mass, elastic stiffness properties, and critical strength ratio are compared for various optimal flexbeam design shapes. The statistical results related to the optimization, which include objective function, constraint violation, feasibility of the solution, number of iterations, number of function evaluations (FE analyses), and CPU time, are also presented. Three independent runs are performed for each of the optimization cases. Only the best solutions are reported here. The maximum number of function evaluations are limited to 5000. The computations are carried out on a machine with 8-core Intel i7 CPU with 8 GB memory.

The properties and strengths of glass-epoxy composite material [13] used in the present study are given in Table 1. The flexbeam sectional design load cases corresponding to various advance ratios (μ) at hover and forward flight conditions are given in Table 2, where F_x is the extensional force, M_x is the torsional moments, and M_y and M_z are the flap and lag bending moments, respectively.

Table 1. Material properties of glass-epoxy [12]

Properties	Values	Strengths	Values
E_1 (GPa)	43	X_T (MPa)	1280
$E_2 = E_3$ (GPa)	8.9	X_C (MPa)	690
$G_{12} = G_{13}$ (GPa)	4.5	Y_T (MPa)	49
G_{23} (GPa)	1.7	Y_C (MPa)	158
$\nu_{12} = \nu_{13}$	0.27	S (MPa)	69
ν_{23}	0.4		
ρ (kg/m^3)	2000		

Table 2. Flexbeam sectional design load cases

Properties	$\mu = 0$ (hover)	$\mu = 0.18$ (forward)	$\mu = 0.30$ (forward)	$\mu = 0.36$ (forward)
F_x (N)	1.89E+05	1.89E+05	1.89E+05	1.91E+05
M_x (Nm)	2.33E+02	2.23E+02	2.14E+02	1.83E+02
M_y (Nm)	-2.26E+02	-1.85E+02	-1.51E+02	-1.15E+02
M_z (Nm)	-6.52E+03	-5.44E+03	-4.14E+03	-2.53E+03

3.1 FE analysis of baseline flexbeam section

The FE analysis of the baseline flexbeam section is first carried out using Ksec2d to determine the sectional properties. The section is considered to be symmetric and does not exhibit elastic couplings. The section is discretized using a mixed mesh consisting of both 8-node quadrilateral and 6-node triangular elements, resulting into 1753 elements and 5064 nodes, as shown in Fig. 4. The computed sectional prop-

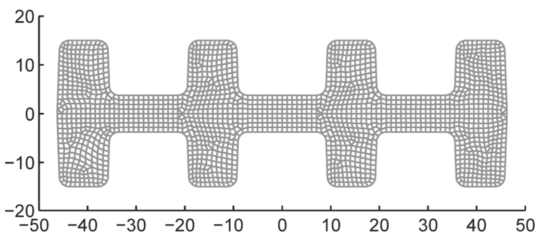

Fig. 4. FE mesh of baseline flexbeam section

Table 3. Computed properties of baseline flexbeam section

Properties	Values
m_0 (kg/m)	3.2092
EA_0 (N)	6.8998E+07
EI_{y0} (Nm^2)	3.8507E+03
EI_{z0} (Nm^2)	5.8157E+04
GJ_0 (Nm^2)	2.5089E+02
R_{cr}	1.3940

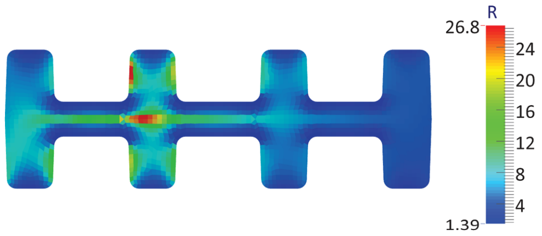

Fig. 5. Strength ratio contour of baseline flexbeam section

erties, which include mass per unit length, extensional, bending and torsional stiffnesses, and critical strength ratio for all design load cases, are presented in Table 3. The contour of the strength ratio at hover condition, which represents the critical value, is shown in Fig. 5.

3.2 Shape optimization

Three section shapes are explored by varying the number of control points of the quadratic B-spline: (a) a H-shape section with 8 control points and 14 design variables, (b) a double-H shape section with 12 control points and 22 design variables, and (c) a triple-H shape section with 16 control points and 30 design variables. The parametric models with the locations of control points are shown in Fig. 6. The *y*- and *z*-locations of the control points comprise the design variables which are considered as integer multiples of 0.1 mm to allow manufacturability.

The optimal flexbeam section shapes for each case are illustrated in Fig. 7. In order to realize the multiple design objectives while satisfying the given constraints, the initial shapes evolve to innovative optimal shapes indicating significant alterations from the baseline. The non-dimensional sectional properties of each of the shapes are compared in Table 4 and the corresponding statistical results are reported in Table 5.

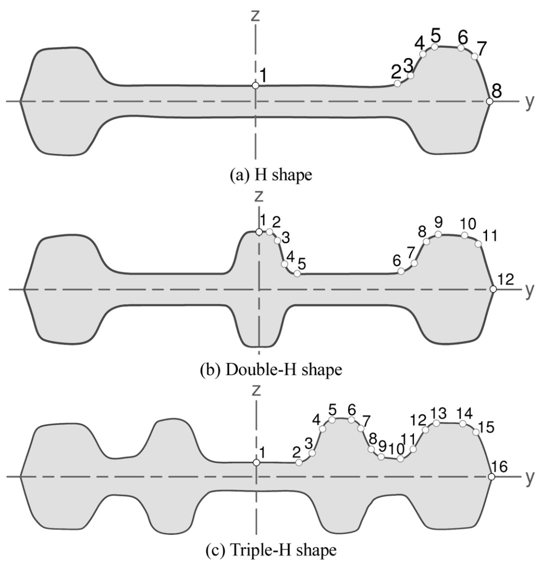

Fig. 6. Parametric models of flexbeam section shapes

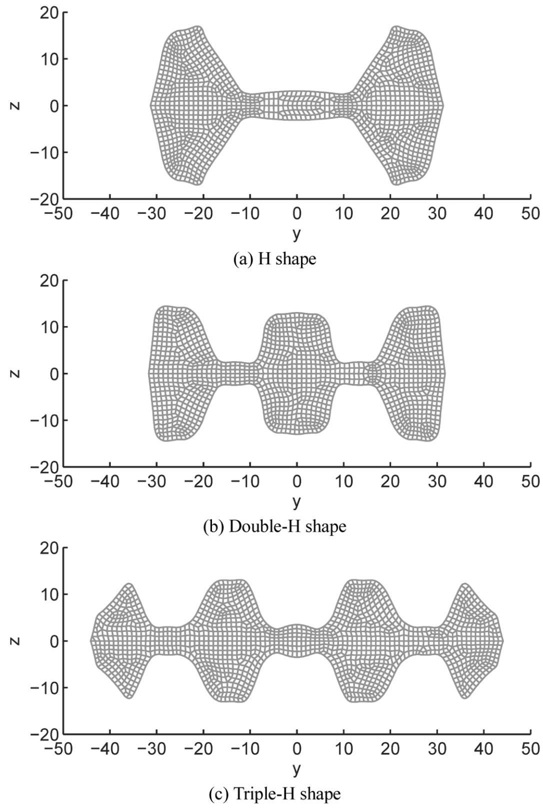

Fig. 7. Optimal flexbeam section shapes

The best value of the primary objective i.e., torsional stiffness is achieved by the double-H shape which offers a reduction of 9.46%. The triple-H shape also records a reasonable reduction of 8.25% in the torsional stiffness whereas the H shape shows negligible change compared to that of the baseline. All the three shapes report higher values of critical strength ratio than the baseline value of 1.3940 since a constraint was imposed for it to be higher than 1.5. The double-H shape also presents the least value of the lag bending stiffness, which is 32.56% of that of the baseline, among all the three shapes. The last objective of low mass is best accomplished by the H shape which shows 35.55% reduction followed by the double-H shape section with 30% reduction from that of the baseline. It is remarked that all the three shapes reach the optimal solutions without violating any of the constraints, as can be seen from Table 5. The double-H shape offers a best solution because of the locations of control points which make it suitable for low torsional and lag bending stiffnesses which are the main design objectives. The mass and the critical strength ratio of double-H shape are also reasonable compared to the other two, given that the favorable differences from the baseline are considerably high. Comparing the efficiency, double-H shape section reaches the feasible optimum in 27 iterations requiring 504 function evaluations which are much less than those needed for H or triple-H shapes. Furthermore, the optimal double-H shape shown in Fig. 7b is much smoother than the other two

Table 4. Comparison of sectional properties for optimal flexbeam shapes

Properties	H shape	Double-H shape	Triple-H shape
m/m_0	0.6445	0.7000	0.7827
EA/EA_0	0.6445	0.7000	0.7827
EI_y/EI_{y0}	0.6658	0.6154	0.4666
EI_z/EI_{z0}	0.3445	0.3256	0.5912
GJ/GJ_0	0.9960	0.9054	0.9175
$R_{cr}/1.5$	1.1044	1.0157	1.0370

Table 5. Comparison of statistical results for flexbeam shape optimization

Parameters	H shape	Double-H shape	Triple-H shape
$f(\mathbf{x})$	0.2004	0.1926	0.2524
$\Phi(\mathbf{x})$	0.0000	0.0000	0.0000
Feasibility	Feasible	Feasible	Feasible
Iterations	44	27	143
No. of evaluations	827	504	2704
CPU time (s)	3141.75	2081.35	8927.45

(Figs. 7a and 7c) without any kinks or dents which renders it viable for manufacturing. The present optimization study thus suggests a simpler and better section achieving multiple design objectives efficiently with double-H shape compared to the slightly complex baseline section with triple-H shape.

4. CONCLUSIONS

In the present work, a section optimization framework is developed using a FE sectional analysis Ksec2d integrated with the optimization algorithm PSGA for the shape optimization of flexbeam sections. The various optimal shapes are explored to achieve the multiple target objectives while satisfying the given constraints. Following conclusions can be drawn from the present study:

(1) The structural shape optimization of open flexbeam sections is successfully carried out achieving multiple design objectives using weighted formulation without violating the constraints.

(2) The feasible optimal section with double-H shape presents a much better solution in comparison to the baseline with lower torsional and lag bending stiffness. The mass reduction as well as the increase in critical strength ratio are also significant compared to those of the baseline.

(3) The optimal solution is reached with few function evaluations which can be attributed to the efficient gradient-free global optimization algorithm PSGA.

(4) The developed section optimization framework demonstrates its capability to explore better designs for arbitrary open and closed section composite beams including helicopter/wind turbine rotor blades.

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