論文

복합재료 구조물의 파괴에 대한 치수효과의 중요성

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The Importance of Size/Scale Effects in the Failure of Composite Structures

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ABSTRACT

In this paper, the importance of the size effects on the strength ratio is demonstrated by numerical results. The rate of decrease of tensile strength is for glass fiber, based on the experience of a composite manufacturing specialist. For other material, similar procedure may be used until detailed test result on such material is available. The strength criterion used is that of Tsai-Wu for stress space. The factors influencing the ratio are, reducing the tensile strength alone or both tensile and compression strengths, selection of the normalized interaction term, that is, the generalized von Mises criterion or the Hill's criterion, and the status of applied stresses. Some of the numerical results are presented for a guideline for the future study.

초 록

이 논문에는 구조물 강도에 대한 치수효과의 중요성이 수치해석의 결과로 강조되어 있다. 유리섬유 인장강도의 감소율은 어느 복합재료 제작전문가의 경험에 의한 것이다.

다른소재의 경우는, 이러한 소재에 대한 자세한 실험결과가 나올 때 까지, 이 논문의 자료와 방법을 사용할수 있다. 강도범주는 Tsai-Wu의 응력공간에 대한 것을 사용하였다. 강도 비율을 지배하는 요소는, 인장강도만 감소시키는 경우, 인장 및 압축강도를 감소시키는 경우, 일반화된 von Mises의 파괴개념 또는 Hill의 개념을 선택하는 경우, 작용하는 응력상태 등이다.

Key Words: 치수효과(size effects), 복합재료구조(composite structures), 강도기준(strength criteria)

1. Introduction

In composite structures, reasonable theory of size/scale effects on the failure mechanism is still lacking. Reduction in fiber strength is experienced when the size of the structures

fiber bundle increases. There are several causes for such decreases of strength, but the diagram showing the rate of tensile strength reduction based on a composite manufacturing specialists experience on filament winding gives one a general guideline to estimate the "apparent" strength value of glass fibers used for filament winding. One can use strength theories with such reduced strength to obtain the failure/strength criteria of the structure. The criterion recommended is that of Tsai-Wu for the stress space. Even though the size effect of a structure is caused by several reasons such as the type of materials, manufacturing methods

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and others, the procedure and the equation given by this paper will give a guideline to estimate the strength and to develop theories for the size effect of a structure made of other than glass fibers and manufactured by methods other than filament winding.

2. Size/Scale Effects on the Failure of Composite Structures

Size effects influence the material Properties of quasi-brittle materials (e.g. concrete and rocks). In case of any material, the larger the volume the greater is the probability of larger flaws. More recently, the mechanics of materials were studied at various scales ranging from atomic scale to microns to large macro or structural behavior. It has been known that linear elastic fracture mechanics (LEFM) applied to lavoratory size quasi brittle materials underestimates fracture toughness. Classical LEFM technique may underestimate the true toughness of certain quasi brittle materials such as geomaterials by as much as an order of magnitude, especially for those with large scale heterogeneities, and using typical laboratory size specimens. The question remains as to how laboratory tests could produce a toughness value closer to the in situ true fracture toughness. We can either build a huge laboratory and test huge specimens: or we can abandon the concept of LEFM. In composite structures reasonable theory of size/scale effects on the failure mechanism is still lacking. Reduction in fiber strength is experienced when the size of the structures fiber bundle increases.

An efficient method to characterize the relationship between strength distribution and size in composites is not complete yet. It has been known that large composites are generally weaker than small composites. There could be several reasons for such phenomenon. One of the most important causes is the scale effect in brittle reinforcing fibers. Brittle fibers are generally strong and uniform in diameter but have the possibility of containing flaws with different strength. A long fiber may have more of such possibility than a short fiber.

Based on the experience of a composite manufacturing specialist, the rate of decrease of tensile strength of glass fibers used for filament wound tubes as the mass of fibers increases is as shown in Figure 1. From the test result reported by Crasto and Kim [5], an approximate relation between 90° tensile strength reduction rate, y, and the volume (proportional to the mass), for the unidirectional composites of AS/3501-6, can be expressed as Figure 2.

Unless there is the test result for the same matrix to be used, this result for epoxy can be used to estimate the rate of the decrease of 90o tensile strength. For each of the constituent materials, both fibers and matrices, the rates of decrease of strength, X, X′, Y, Y′, and S, as the mass increases, must be obtained in the future. The manufacturing method and other possible factors also have to be considered. Any strength theory can be used with the "reduced" strength as given above.

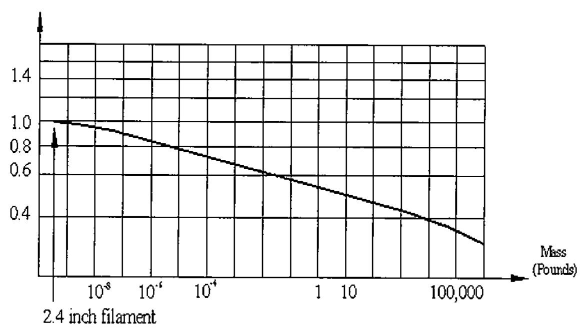

Fig. 1 Rate of decrease of glass fiber tensile strength based on mass (Courtesy of Mr. J . Lowrie Mclarty).

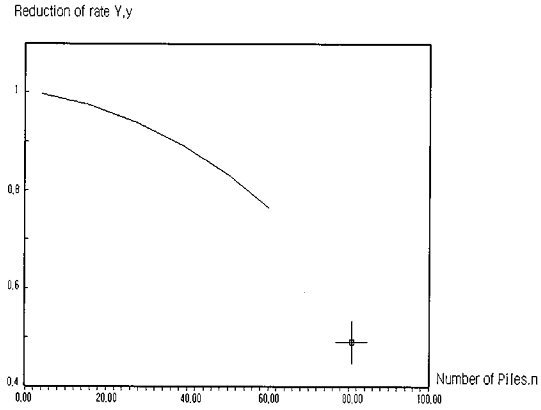

Fig. 2 Tensile strength reduction rate of epoxy matrix based on mass.

3. Strength Theories

3.1 Maximum Strength Theory

Jenkins extended the concept of the maximum normal or principal stress theory to predict the strength of planar orthotropic materialls such as wood. According to this theory, failure will occur when one or more than one of the stresses acting into the directions of material symmetry, σ_1, σ_2 and τ_{12}, reaches a respective maximum value, X, Y, and S. Mathematically stated, failure will not occur as long as

$$
\begin{aligned}
&-X' < \sigma_1 < X, \\
&-Y' < \sigma_2 < Y, \text{ and} \\
&-S' < \tau_{12} < S.
\end{aligned}
\tag{1}
$$

Because of orthotropic symmetry, the shear strength is independent of the sign of τ_{12}. There are five independent modes of failure, and there is no interaction among the modes according to this theory. The reality is that failure processes are highly interacting and far more complex than the values of the stress components. If the stress ratio is used, this criteria can be expressed as

$$
\begin{aligned}
&R_x = X/\sigma_x, \text{ if } \sigma_x>0, \text{ or} \\
&R_x' = X'/|\sigma_x|, \text{ if } \sigma_x<0, \\
&R_y = Y/\sigma_y, \text{ if } \sigma_y>0, \text{ or} \\
&R_y' = Y'/|\sigma_y|, \text{ if } \sigma_y<0, \text{ and} \\
&R_s = S/|\sigma_s|, \text{ if } \sigma_s<0.
\end{aligned}
\tag{2}
$$

3.2 Maximum Strain Theory

The maximum strain theory is an extension of the maximum principal strain theory, promoted by Poncelet and Saint-Venant, to anisotropic media. The strain components for an orthotropic lamina are referred to the principal material axes, and there are three strain components in this criterion. Since linear elastic response is assumed to failure, this criterion can predict strength in terms of loads or stresses. A ply of a laminate is considered failed when one of ε_x, ε_y, and ε_s reaches the maximum value obtained from simple one-dimensional testing. This maximum strain obtained from each test is either measured or computed from the measured strength divided by the tangent modulus:

$$
\begin{aligned}
&\varepsilon^*_x = X/E_x, \text{ or } \varepsilon'_x = X'/E_x, \\
&\varepsilon^*_y = Y/E_y, \text{ or } \varepsilon'_y = Y'/E_y, \text{ and} \\
&\varepsilon^*_s = S/E_s.
\end{aligned}
\tag{3}
$$

The minimum common envelope of the superposition of the interaction failure diagrams, for either stress or strain, of all individual plies, becomes the failure diagram for the laminate. The strength ratio is expressed by the lowest of three ratios of the maximum strain to the applied strain. Note similar procedure taken for the maximum stress criteria,

$$
\begin{aligned}
&R_x = \varepsilon^* x/\varepsilon_x, \text{ if } \varepsilon_x>0, \text{ or} \\
&R'_x = \varepsilon^{*\prime} x/|\varepsilon_x|, \text{ if } \varepsilon_x<0, \\
&R_y = \varepsilon^* y/\varepsilon_y, \text{ if } \varepsilon_y>0, \text{ or} \\
&R'_y = \varepsilon^{*\prime} y/|\varepsilon_y|, \text{ if } \varepsilon_y<0, \text{ and} \\
&R'_s = \varepsilon^*_s/|\varepsilon_s|.
\end{aligned}
\tag{4}
$$

3.3 Review of Failure Theories

Both the maximum stress and maximum strain criteria assume no interaction among the possible five modes. Since the Poisson's ratio is not zero, there is always coupling between the normal components, and this leads to disagreement between these two criteria regarding the magnitude of the load and the mode for the failure.

For example, consider a unidirectionally reinforced laminate acted upon by uniaxial tension, σ, at some angle θ to the reinforcements. The maximum allowable loading is the smallest of the following equations :

1) From the maximum stress theory,

$$
\sigma = \frac{X}{\cos^2\theta}, \quad
\sigma = \frac{Y}{\sin^2\theta}, \text{ or} \quad
\sigma = \frac{S}{\sin\theta\cos\theta}.
\tag{5}
$$

2) From the maximum stain theory,

$$
\sigma = \frac{X}{\cos^2\theta - \nu_{12}\sin^2\theta}, \quad
\sigma = \frac{Y}{\sin^2\theta - \nu_{21}\cos^2\theta}, \text{ or} \quad
\sigma = \frac{S}{\sin\theta\cos\theta}.
\tag{6}
$$

The result of two criteria agrees only on the shear plane and along the four lines of constant failures due to uniaxial stresses. Just as the deformation of a body is always coupled by the nonzero Poisson's ratio, failure of a body is also coupled. Because the micromechanics of failure is highly coupled, we should not extend the simple failure modes, based on maximum stress or maximum strain components, to fiber, matrix, and interfacial failure modes.

3.4 Tsai-Wu Strength Criterion in Stress Space

The strength ratio, R. is the ratio of the maximum or ultimate strength to the applied stress. The definition of the strength ratio indicates that

$$\{\sigma\}max = R\{\sigma\}_{applied}, \text{ and} \quad (7)$$
$$\{\varepsilon\}max = R\{\varepsilon\}_{applied}.$$

The R is analogous to the safety factor or the load factor : Failure occurs when R = 1.

When R < 1, the applied stress is larger than the strength by a factor of 1/R. This is physically impossible but provides a useful information for design. For example, one may reduce the applied load by (R − 1).

According to Tsai [1], an easy way to incorporate a coupled or interacting failure criterion is to use the quadratic criterion. This is a generalization of strain or distortion energy, proposed by Maxwell, and further developed by Huber. By using this criteria, we can recognize failure criteria as useful design tools on fitting available data, instead of depending on failure criteria to define the modes of failure. Tsai and Wu assume that the criterion in stress space is the sum of linear and quadratic scalar products;

$$F_{ij}\sigma_i\sigma_j+F_i\sigma_i=1, \quad (8)$$
$$i,j=1,2,3,4,5,6.$$

The F_i and F_{ij} are second and fourth order lamina strength tensors. The linear stress terms are count for possible differences in tensile and compressive strengths. The quadratic stress terms describe an ellipsoid in stress space. The F_{ij} ($i\neq j$) terms are new. Off-diagonal terms of the strength tensor represent independent interactions among the stress components.

For a thin orthotropic ply under plane stress status relative to the symmetric axes x-y, this failure criterion becomes,

$$F_{xx}\sigma_x^2+F_{yy}\sigma_y^2+2F_{xy}\sigma_x\sigma_y+F_{ss}\sigma_s^2+F_x\sigma_x+F_y\sigma_y+F_s\sigma_s=1, \quad (9)$$

where the strength parameters, Fs, can be obtained from

$$F_{xx}=1/XX', \quad F_{yy}=1/YY', \quad F_{ss}=1/S^2, \quad (10)$$
$$F_x=1/X-1/X', \quad F_y=1/Y-1/Y', \quad Fs=0,$$

where

X : Longitudinal (or uniaxial) tensile strength,

X' : Longitudinal (or uniaxial) compressive strength,

Y : Transverse tensile strength,

Y' : Transverse compressive strength,

S : Longitudinal shear strength.

4. Recommended Strength Failure Analysis Procedure

With available information at present, the following strength failure analysis procedure is recommended for glass fiber reinforced composites with epoxy matrix.

1. Obtain reduced X by Fig. 1
2. Assume the scale effect is the same for both tension and compression. (This assumption may be corrected when detailed research result is available).
3. Obtain reduceel Y by Fig. 2
4. Assume Y=Y'.
5. Assume S=S' (Coupon).
6. Use Tsai-Wu failure criteria for stressspace. Since the rates of decrease of the moduli are not known, use of the criteria for strain space is complicated.

The strength obtained by the above steps may not be "exact" for the composite with a given "increased" size. However, the result should not be too far off. Something is always better than nothing. Using strength theory with reduced tensile strength value alone is far better than designing the structure with the coupon test values. The recommended procedure will result in safer structures and will accelerate further studies for the exact failure-strength theories for composite structures with different scales/sizes, and with various constituent materials. When materials other than glass fibers and epoxy are used, only Fig. 1 and 2 may be modified.

When detailed information on size effect for materials other than glass fibers and epoxy is not known, one can use Fig. 1 and 2 given above.

5. Numerical Example

The structure under consideration is the pressure pipe as shown in Fig. 3

Internal diameter : 4 m,

Thickness of the pipe : 0.031 m,
Maximum operating pressure : 2 MPa,
Design tensile strength of circular ply material :352 MPa,
Design tensile strength of longitudinal ply material : 352 MPa,
Wall thickness h=248 mm, h_0=0.125 mm.

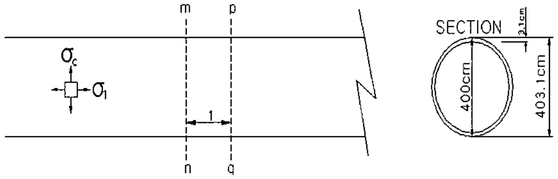

Fig. 3 The structure under consideration.

Stresses due to the internal pressure,

$$\sigma_c = \frac{PD}{2t} = \frac{2\times 4}{2\times 0.031} = 129MPa,$$

$$\sigma_l = \frac{PD}{4t} = \frac{2\times 4}{4\times 0.031} = 64.5MPa,\ \sigma_s = 0.$$

5.1 Safety factor, R, without size effect considered

1) Ordinary strength theory

$\sigma_{max} = R\sigma_{applied}$,
$\sigma_c(max) = R_c\sigma_{applied}$,
$\sigma_l(max) = R_l\sigma_{applied}$,
R_c = 2.7287, R_l = 5.4574.

2) Tsai-Wu failure criteria considering tensile and compression strengths only

	$F_{xy}^* = 0$	$F_{xy}^* = -1/2$
R	2.4340	3.1404

5.2 Safety factor, with size effect considered

Assuming the filament diameter nomenclature as J, one ply of h_o=0.000125m has about 10 fiber diameter thickness. With V_f =0.45, one ply has about 5 fibers through its thickness.
Volume of fibers=

$$(5\times 248)\times(\frac{100}{2.54\times 2.4})\times(\frac{2\times 3.14\times 200}{0.125}) = 216,537,728$$

times of one fiber.
From Figure 1, the stress reduction ratio of fiber is 0.59, and that of the matrix is 0.71.
Thus

1) Ordinary strength theory

$\sigma_{max} = R\ \sigma_{applied}$,
$\sigma_{c(max)} = R_c\sigma_{applied}$,
$\sigma_{l(max)} = R_l\sigma_{applied}$.
R_c = 1.6061, R_l = 3.875.

2) Tasi-Wu failure criteria considering tensile and compression strengths only

	$F_{xy}^* = 0$	$F_{xy}^* = -1/2$
R	1.5699	1.9713

5.3 Comparison

1) Ordinary strength theory

	R_c	R_l
Without size effect considered	2.720	5.459
Size effect considered	1.6061	3.875

2) Tasi-Wu failure criteria considering tensile and compression strengths only

	$F_{xy}^* = 0$	$F_{xy}^* = -1/2$
Without size effect considered	2.4340	3.1404
Size effect considered	1.5699	1.9713

The senior author, in his previous papers [2-4], proposed the strength failure analysis procedure considering size effect, and conclouded that the strength ratio depends on five factors : two cases of test coupon strengths, that is, A) reduction is made for both tensile and compression, B) reduction is made for tensile strength only, two failure criteria, F_{xy}^*= 0 and F_{xy}^*= -1/2, and the status of the applied stress. The proposed R/D direction on size /scale effect, then, can be summarized as follows.

A. Obtain the rate of decrease of the fiber str-ength based on mass, for each of the possible candidate materials for large size structures.
B. Same as A for matrix.
C. For each of the laminate type to be used for design, perform tests, under all possible combination of the applied stresses.
D. With the result of above A, B and C, find out which one of the failure criteria, F_{xy}^*= 0 and F_{xy}^*= -1/2, is closer to the test result, for each combination of the stresses.
E. With the result of A, B, and C, find out whether reduction of the transverse strength is significant or

not, for each state of the stresses.

F. Find out whether reduction should be made for both tensile and compression strengths or tensile strength only, for each state of the stresses.

6. Conclusions

In this paper, the importance of size effect on strength of a composite is demonstrated by numerical examination. The Tsai-Wu failure criterion for stress space is used since the rates of decreases of moduli are not known, if strain space is used. It is shown that strength reduction is necessary for safe design of a structure. The strength ratio, R, is a function of five factors : two cases of test coupon stre-ngths, A) and B), two failure criteria, $F_{xy}^* = 0$ and $F_{xy}^* = -1/2$, and the status of applied stresses. The proposed procedure in this paper is based on glass fibers and epoxy matrix. This procedure can be used for composites with other constituent materials. As further studies are made on such materials, only Figures.1 and 2 can be modified. Something is always better than nothing.

The effect of the size/scale may be very serious. The numerical example in this paper shows that the safety factor is between 5.459 and 1.5699.

Acknowledgements

The glass fiber tensile performance data based on the mass under tension is provided by one of the pioneer composite manufacturing specialists, Mr. J. Lowrie Mclarty.

This paper was supported by the research tund of Seoul National University of Technology.

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