

論文

설계변수에 대한 샌드위치 보의 파손하중

김종만^{*+}

Influence of Design Variables on Failure Loads of Sandwich Beam

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ABSTRACT

Sandwich structures have been widely used in the applications of vessel industry, where high structural stiffness is required with small addition of weight. It is so significant to think of the effect of the variables in the design process of the sandwich structure for the concentrated loads. This paper describes the influence of design variables, such as core density, core thickness and face thickness ratio, on the strength of sandwich beam. The theoretical failure loads based on the 2-D elasticity theory agree well with the experimental yield or failure loads, which are measured at the three point bending laboratory test using AS4/3501-6 facing and polyurethane foam core sandwich beam. The comparison of those yield or failure loads was also done with the ratio of the top to bottom face thickness. The theoretical optimum condition is obtained by finding the intersection point of failure modes involved, which gives optimum core density of the sandwich beam for strength and stiffness. In the addition, the effect of unequal face thickness for the optimized and off-optimized sandwich beams for the strength was compared with the ratio of loading length to beam length, and the variations of strength and stiffness were discussed with the relative ratio of core to face mass.

초 록

샌드위치 구조물들은 적은 무게의 첨가로 높은 강성(stiffness)을 요구하는 조선업에 널리 사용 되어져 왔다. 국부하중 조건 하에서 샌드위치 구조물에 대한 디자인 변수들을 고려하는 것은 중요시되어졌다. 이 연구는 샌드위치 보의 강도에 대한 core층의 밀도, core층의 두께 그리고 face층의 두께 비율의 영향을 기술하였다. 이차원 탄성이론에 바탕을 둔 파손 하중은 AS4/3501-6 facing과 polyurethane foam core 샌드위치 보의 3점 굴곡 실험 결과와 잘 일치 하였다. 또한 그러한 파손 하중들은 face층의 비율의 변화와 함께 비교되었다. 파괴 mode들의 교차점으로 결정되어진 최적조건은 강도(strength)와 강성(stiffness)에 대한 샌드위치 범의 최적 core 밀도의 값이 결정되었다. 추가적으로 강도에 대한 최적조건과 그렇지 못한 샌드위치 보에 대한 face 두께 비율 효과가 하중 길이에 따라 비교되었으며, 강도와 강성이 core/face 무게 비율과 함께 검토하였다.

Key Words: 샌드위치 구조물(Sandwich structure), 샌드위치 보(Sandwich beam), 파괴 분석(Failure analysis), 최적화(Optimization)

1. Introduction

Many sandwich constructions have been widely used in the naval vessels and vehicles[1], where are needed high stiffness

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and strength to weight ratios. The sandwich effect of increasing the second moment of area makes stronger and stiffer materials, such as carbon fiber, Kevlar and aluminum alloy, used as thin face layers and balsa wood, expanded polymer foams, synthetic foams, and honeycombs, be employed as a core layer. The face and core layers are firmly bonded with many excellent structural adhesives.

The endeavor of improving the stiffness and strength to weight ratio on the sandwich beam requires more accurate theoretical analysis to predict the load-carrying capability of the structure. However, it is so complicated as not to describe the high stress and strain concentration in the sandwich structures due to the localized loads by simpler theories. An improvement in the analysis of sandwich beam has been introduced by the higher order theory of Frostig et al[2-4], where each face is treated as being independent beam and core includes shear deformation and finite stiffness in the through-the thickness direction. The assumption of the constant shear stress through the thickness of the core in the higher order theory does not seem correct in the neighborhood of the concentrated loading region. An elasticity solution, originally developed by Pagano[5] showed a good comparison with higher order theory and finite element analysis[6]. The 3-D elasticity solution for layered orthotropic plate was employed to the modeling of 2-D sandwich beam for concentrated loads[6]. Though this solution is restricted to the simply supported boundary condition, it can be applied to the structures with any numbers of layers and easily adapted for its implementation. The beam bending tests were performed for the sandwich beam specimens with the simply supported end conditions, which could be used for the results applicable for the quasi-static impact.

Recent developments on the mechanical properties of foam materials have given the significant information to the theoretical analysis and its optimization for stiffness on the sandwich structures[7]. An analytical method for finding the value of core density, core thickness, and face thickness, which minimize the weight of the sandwich beam of given stiffness, has been reported[8].

In this paper, the prediction of failure loads is carried out by 2-D specialized form of the elasticity solution and compared with the experimental yield loads, which are determined by reading the loads at the intersection point of the slope lines between the linear region and nonlinear region because the sandwich structure generally shows nonlinear behavior above a certain range in the typical load-displacement curve. Based on the good agreements

between the theoretical prediction and experimental results in the linear region[9], the optimization of sandwich beam for the strength is worked[10]. Further research is working on the nonlinear response of foam core sandwich beam. The effects of core density and face thickness are investigated with optimized and off-optimized beams. These kinds of information will give invaluable information on the design of sandwich beam for high ratio of strength to weight under the concentrated loads.

2. Theoretical Background

As mentioned in the introduction, 2-D elasticity analysis of the sandwich beam for the concentrated loads was employed from the 3-D elasticity solution of Pagano[5] and was used for the failure analysis and its optimization for the strength. This solution is not restricted to the specific material properties and the number of layers. However, the restriction of simply supported boundary condition is applied to this elasticity solution. The elasticity analysis had the loading spread uniformly over the length equal to 0.65 times the top face thickness, to avoid the singularity associated with a point load. The detailed equations and solution procedures are mentioned in the Reference[10].

The significance of this elasticity theory for the concentrated loading was shown in the comparison with first order shear deformation theory[10]. Though this solution is limited by linear elastic range and failed to predict ultimate failure loads, it will give important insight in the design of sandwich beam for high ratio of strength to weight under the concentrated loads.

3. Experimental

Two kinds of densities in the polyurethane foam (160 kg/m^3 , 320 kg/m^3) with the thickness of 3.18 mm, 6.35 mm and 12.7 mm from General Plastics Co. were chosen as core material and $[0_2/90_2]$, AS4/3501-6 carbon/epoxy was used for each face. Each face was bonded with a core layer by using Hysol EA 9309NA for 24 hrs at room temperature with a slight pressure of 0.14 MPa. Panels were constructed using standard processing techniques in a hot press. Before making the sandwich beam specimen, the characterizations of the polyurethane foam were performed to measure its compressive yield properties and its shear properties according to the ASTM procedures[11].

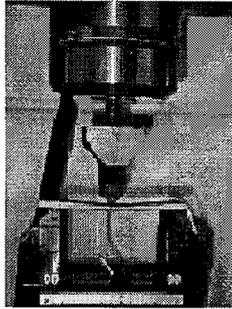


Fig. 1 Three-point bend test of sandwich beam specimen.

The sample dimension is 25.4 mm in width by 203 mm in length, and the specimens were cut from the panels. The span length of 3-point bending test is 152.4 mm, and the support and loading pins had a diameter of 6.35 mm as shown in Fig. 1. Three-point bend tests of the specimens were performed, according to the ASTM D 790. Load was applied by the MTS 880 machine with a constant stroke rate of 1.27 mm/min, and was recorded by the data acquisition function of MTS 880 machine.

4. Effects of Core and Face Properties

The variables considered in the optimized design of sandwich structures for stiffness are core density, core thickness and face thickness. In this paper, the effect of those parameters on the failure loads of the sandwich beams under the concentrated loading will be discussed in the following.

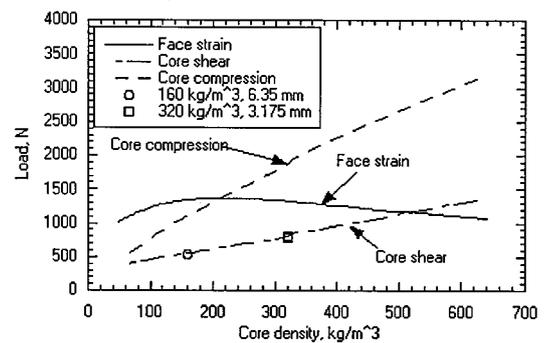
4.1 Effect of Core density

In the elasticity solution analysis, three failure modes are considered, which are compressive face failure, core shear failure, and core failure in compression from the experimental observation of the samples used.

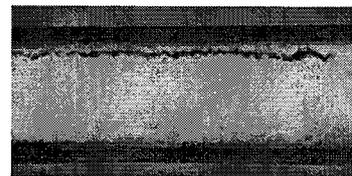
First of all, face failure is assumed to occur if the critical compressive strain reaches a value of 1.4 %, which is equal to the critical strain in tension, for the AS4 carbon fiber on the basis that the failure strain in compression is lower than it is in tension[12]. Two failure modes were taken from the core layer. One of them is core shear failure. The other one is core failure in compression, which is also described as indentation failure. Each core failure is assumed to occur if the maximum yield stresses in the core reach its own compressive or shear yield strengths. The failure loads are taken by the failure mode with the lowest yield load.

Predicted yield loads show good agreements with the experimental yield loads in Fig. 2. The relative optimum yield load is obtained at the core density of 568 kg/m^3 and the core thickness of 1.81 mm at the constant beam mass of 0.0166 kg. The failure loads of sandwich beam up to 568 kg/m^3 core density are controlled by the core shear failure mode and the failure loads of the sandwich beam more than 568 kg/m^3 core density are dominated by the face failure mode.

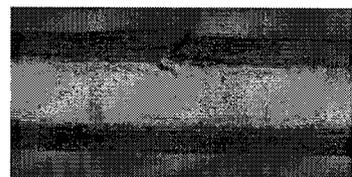
The failed specimen of 160 kg/m^3 core density sandwich beam with 6.35 mm core thickness in Fig. 2b shows the shear failure in core as it is predicted in Fig. 2a. In the failed specimen of 320 kg/m^3 core density with the core thickness of 3.175 mm, both core shear failure and face strain failure are shown in Fig. 2c. Face failure seems to be followed after core shear failure as a secondary failure mode as based on the Fig. 2a.



a) Comparison of measured yield or failure loads with predicted loads and modes.



b) Shear failure mode for 160 kg/m^3 core density.



c) Shear failure and face failure mode for 320 kg/m^3 core density

Fig. 2 The variation of yield load of $[0_2/90_2]$, equal faces thickness sandwich beams with core densities. (Experimental yield loads of 160 kg/m^3 , 6.35 mm and 320 kg/m^3 , 3.175 mm core sandwich beams are added).

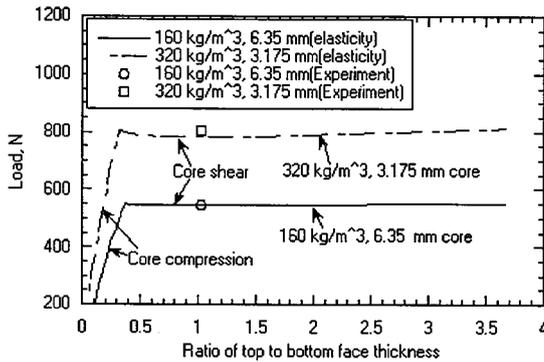


Fig. 3 The comparison of experimental yield or failure loads of 160 kg/m³, 6.35 mm and 320 kg/m³, 3.175 mm core sandwich beams with different ratio of [0₂/90₂]_s faces with the prediction of elasticity analysis.

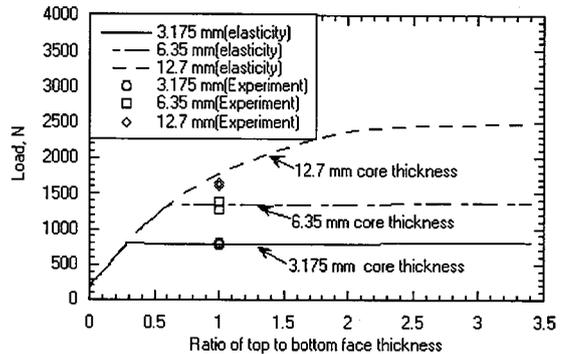


Fig. 4 The effect of core thickness on the yield or failure loads of sandwich beam with the core density of 320 kg/m³ and [0₂/90₂]_s faces.

4.2 Effect of Face Thickness Ratio

The effect of unequal face thickness ratio on the yield or failure loads of sandwich beam with the total 16 ply faces is also investigated with the constant core mass. The failure loads are controlled by core failure mode in compression and core shear failure mode, and the experimentally measured loads are agreed well with the predicted failure loads, as shown in Fig. 3.

The interesting point of the prediction of failure loads is that the failure loads are not improved with the thicker top (loaded) face with respect to the bottom face in the dominated region of core shear mode. The failure loads of 320 kg/m³ core density sandwich beam with the core thickness of 3.175 mm is higher than those of 160 kg/m³, 6.35 mm core sandwich beam, which can be also explained in Fig. 2a.

The effect of top to bottom face thickness ratio on the yield loads at 320 kg/m³ core density sandwich beam is investigated with the experimental failure loads in Fig. 4. An interesting point is that the same slope, as shown in Fig. 4, exists for sandwich beams having the same core densities with different core thickness under the concentrated loads. More than a certain ratio of top to bottom face thickness, which depends on the core thickness, the failure mode dominating the failure loads is changed to the core shear failure mode.

5. The Optimum Core Density and Face Thickness Ratio

It is significant to find the core density and face thickness ratio to the optimum condition for the strength to weight ratio in the design of the sandwich structure. The investigation of the influence of those variables on the failure loads is also important for both optimized and off-optimized beams.

5.1 Influence of Core Density

An optimization of the beam with the respect to the core density is pursued with a certain face mass and an assumed value of the core mass relative to the mass of the two faces and gives the true optimum core density at the intersection point of the three failure modes, which are face failure, core shear failure and core failure in compression.

As an example, the optimization procedure is applied to the sandwich beam with [0₂/90₂]_s equal faces in Fig. 5. The true optimum core density is 456 kg/m³ and the core thickness is 6.91 mm. The optimum ratio of the core mass to face mass is 96.1 %.

The optimum ratio of the core mass to face mass in the design for maximum strength to weight ratio is compared with experimental results in Fig. 6. The used specimens show clear deviation in the selection of the optimum core mass for strength at a given face mass, which explains the reason not reaching the optimum loads for maximum strength. They were not made with the consideration of the optimum condition for the optimum strength to weight ratio before testing.

The true optimum core densities for strength and for stiffness are compared in Fig. 7, indicating that the optimum core density for strength is higher than that for stiffness

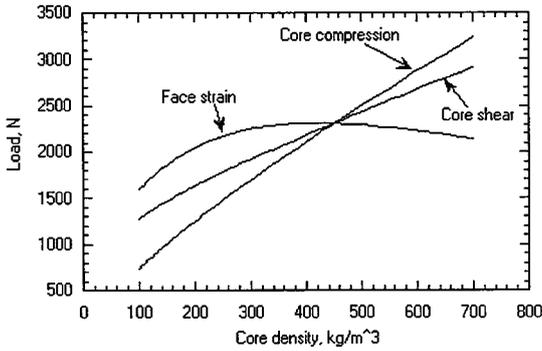


Fig. 5 Predicted yield load for example beam. The intersection of the three failure criteria gives a true optimum design for concentrated loading, which occurs for this example at a core density of 456 kg/m³.

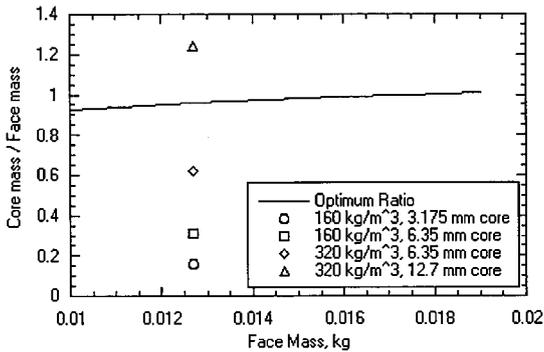
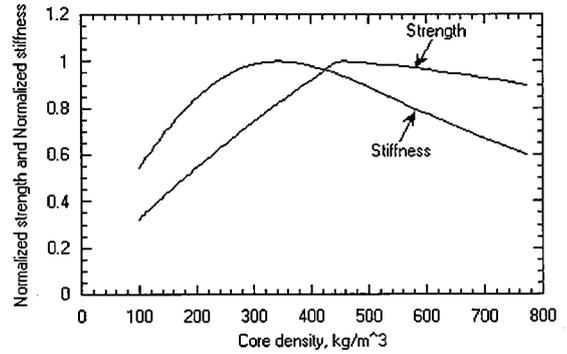


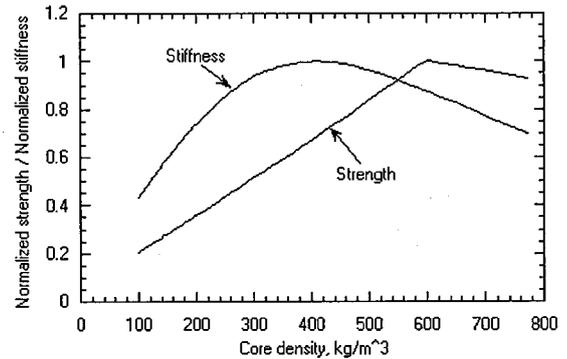
Fig. 6 A comparison between the optimum mass ratio and the mass ratio of the specimens with [0₂/90₂]_x equal faces.

under the concentrated loads, which is the ratio of 0.0045 of the loading length to beam length. To make it clear, a different ratio of loading length to beam length of 0.045 is arbitrarily chosen. The optimum condition for the loading length ratio of 0.0045 is 456 kg/m³ core density with the core thickness of 6.91 mm, and that for loading length ratio of 0.045 is 603 kg/m³ core density with the core thickness of 8.33 mm.

In the point of the sandwich beam for the maximum strength to weight ratio, the optimum core density at the loading length ratio of 0.045 is higher than that at the ratio of 0.0045. An optimum core density for stiffness also requires the higher core density when the loading length is varied from 0.0045 to 0.045. The optimum core density for the maximum strength to weight ratio is higher than about 35 % with the increase of loading length of 10 times, while the optimum core density for stiffness is varied to the higher value of 20 %. It seems that the optimum core density for



a) A ratio of loading length to beam length = 0.0045. Strength is normalized to 2278 N and stiffness to 479 N/mm.



b) A ratio of loading length to beam length = 0.045. Strength is normalized to 3915 N and stiffness to 821 N/mm.

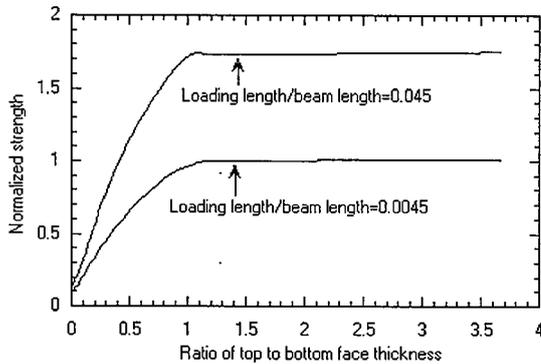
Fig. 7 Predicted effect of varying the core density at different loading lengths. Faces are 8 plies of each of AS4/3501-6, and constant core weight of 160 kg/m³, 6.35 mm.

strength is higher than that for stiffness in the concentrated loading region.

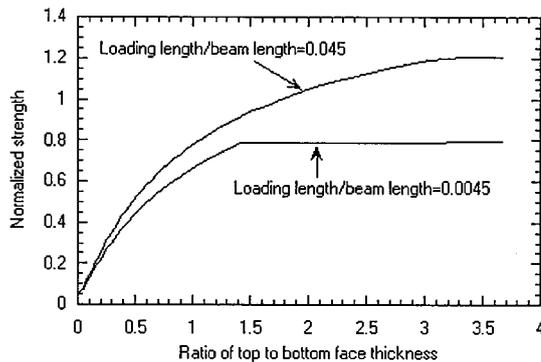
5.2 Influence of Face Thickness Ratio

The strain concentration in the neighborhood of the loading region due to the concentrated loads was explained successfully by the elasticity solution[10]. Therefore, an unequal face thickness for the optimized and off-optimized sandwich beams could be an important parameter to affect the failure loads of the sandwich beam.

At the optimum condition for the strength to weight ratio on the both ratios of loading length to beam length, the effect of increasing the thickness of the top (loaded) face is seen in Fig. 8a. The beam strength is not much improved with the thicker top (loaded) face thickness with respect to the bottom (unloaded) face at the optimum condition.



a) Optimized sandwich beam for loading length ratio of 0.0045(456 kg/m³, 6.91mm core) and of 0.045(603 kg/m³, 8.33 mm core).



b) Off-optimized sandwich beam for loading length of 0.045(260 kg/m³, 12.06 mm core) and of 0.045(260 kg/m³, 19.35 mm core).

Fig. 8 An effect of unequal face thickness in the optimized and off-optimized sandwich beams with the total 16 ply face thickness at the two different concentrated loading lengths. Strength is normalized to 2278 N.

In the off-optimized sandwich beam with 260 kg/m³ core density, which is the same core mass as the optimum condition to each loading length ratio, the variation of failure loads is investigated with the ratio of top to bottom face thickness in Fig. 7b. The thicker loaded face is so effective in the improvement of the strength of the off-optimized sandwich beam.

6. Influence of the Ratio of Core Mass to Face Mass

The strength and stiffness of sandwich beam with 260 kg/m³ core density are investigated with the ratio of core mass to face mass at the both loading length ratio of 0.0045 and 0.045 in Fig. 9. As the loading length ratio is increased

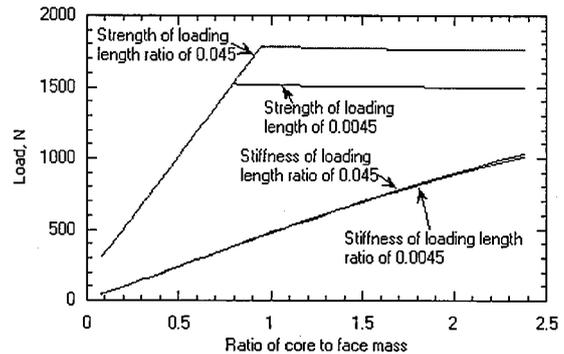


Fig. 9 Variations of the strength and stiffness of the optimized sandwich beams with the different loading length ratios of 0.0045 and 0.045.

from 0.0045 to 0.045, the strength of the beam against the concentrated loads is increased, which indicates less strain concentration in the neighborhood of the concentrated loads. Although the stiffness is increased with the ratio of the core mass to face mass, the stiffness is insensitive to the variation of loading length.

7. Summary and Conclusion

The influences of the core density and face thickness ratio on the failure load of sandwich beam with the 2-D elasticity solution are discussed. Experimentally measured yield or failure loads show good agreements with the predicted failure loads. The reasonable selection of both core density and face thickness ratio based on the optimization procedure could result in the optimum strength to weight ratio of sandwich beam. This paper specially contributes to show the effects of design variables on the sandwich beam for the maximum strength to weight ratio on different concentrated loading zone lengths. The thicker loaded face with respect to the unloaded face could be more effective to resist the localized loads in the off-optimized sandwich beam for the strength. When the sandwich beam is designed for the strength to weight ratio under the concentrated loads, the effects of both core density and face thickness ratio on the failure load should be considered to satisfy the structural requirements.

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