論文

# A Simple Method of Vibration Analysis of Special Orthotropic Plate with A Pair of Opposite Edges Simply Supported and the Other Pair of Opposite Edges Free

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# 양단단순-타단자유인 특별직교이방성 적층복합판의 간편한 진동해석

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## 초 록

이 논문에서는 첨가질량이 있거나 없는 구조요소에 대한 간편하고 정확한 진동해석 방법이 주어진다. 사용된 방법은 1974년 Kim, D. H.에 의해 개발되었다. 이 방법은 변단면과 다양한 경계조건을 갖는 판에 매우 효과적이다. 이 방법은 양단단순-타단자유를 갖는 특별직교이방성 적층복합판에 대해 적용되었다. 이러한 판은 콘크리트와 거더-가로보 시스템을 포함하는 단순 지지된 교량상판의 대부분을 나타낸다. 쉬운 이해를 위해 상세한 실예가 보와 판에 대해 주어졌다.

## ABSTRACT

In this paper, a simple but accurate method of vibration analysis of structural elements with or without attached mass/masses is presented. The method used has been developed by D. H., Kim since 1974. This method is very effective for the plates with arbitrary boundary conditions and irregular sections. This method is applied to the special orthotropic plate with two opposite edges simply supported and the other two opposite edges free. Such plate represents the most of the simply supported bridges/decks, including concrete and girders-cross beam systems. Detailed illustration is given for beams and plates for easy understanding. Some laminate orientation for which the special orthotropic equations can be applied are identified.


## 1. INTRODUCTION

Composite materials can be used economically and efficiently in broad civil engineering applications when standards and processes for analysis, design, fabrication, construction and quality control are established. Many of the bridge systems, including the girders and cross-beams, and concrete decks behave as the special orthotropic plates which have $[0^{\circ}, 90^{\circ}, 0^{\circ}]_r$ fiber orientations.


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Some laminate orientations such as $[\alpha\beta]_r$, $[\alpha\beta\alpha]_r$, $[\alpha\beta\beta\alpha\alpha\beta]_r$, and $[\alpha\beta\beta\gamma\alpha\alpha\beta]_r$ with certain values of $\alpha$, $\beta$, and $\gamma$, and with increasing r, have decreasing values of $B_{16}$, $B_{26}$, $D_{16}$, and $D_{26}$ stiffnesses, where $\alpha$, $\beta$, and $\gamma$ are the fiber orientation in degrees measured from the laminate axes, positive in the counterclockwise direction[1,9,10], r is an integer, and $B_{ij}$ and $D_{ij}$ are the bending-stretching coupling stiffness matrix and the flexural stiffness matrix, respectively. $D_{ij}$ expresses the relation between the stress couples, $M_{ij}$s, and the curvatures, $K_{ij}$s. $B_{ij}$ relates $M_{ij}$s to the mid-surface strains, $\varepsilon_{o\ ij}$s and the in-plane stress resultants, $N_{ij}$s, to $K_{ij}$s. $B_{16}$ and $B_{26}$ cause bending-shearing and stretching-twisting coupling. $D_{16}$ and $D_{26}$ cause bending-twisting coupling. Such laminates given above may be very useful when one tries to apply the advanced composite materials to new constructions such as building slabs, bridge decks, and so on. He can obtain the advantages of the advanced composite materials using simplified equations. For such laminates, the simple equations for the special orthotropic plates can be used[1].

In case of a laminated composite plate with boundary conditions other than Navier or Levy solution types, or with irregular cross section, or with nonuniform mass including point masses, analytical solution is very difficult to obtain. Numerical method for eigenvalue problems are also very much involved in seeking such a solution[8,9,10].

The basic concept of the Rayleigh method, the most popular analytical method for vibration analysis of a single degree of freedom system, is the principle of conservation of energy ; the energy in a free vibrating system must remain constant if no damping forces act to absorb it. In case of a beam, which has an infinite number of degrees of freedom, it is necessary to assume a shape function inorder to reduce the beam to a single degree of freedom system[11]. The frequency of vibration can be found by equating the maximum strain energy developed during the motion to the maximum kinetic energy. This method, however, yields the solution either equal to or larger than the real one. Recall that Rayleigh's quotient $\geq 1$ [1] (pp.189-191). For a complex beam, assuming a correct shape function is not possible. In such cases, the solution obtained is larger than the real one.

A method of calculating the natural frequency corresponding to the first mode of vibration of beam and tower structures was developed and reported by Kim, D. H. in 1974[2]. In this report, the effect of neglecting the weight of beams on the natural frequency is given for several beam support types.

Recently, this method was extended to the first mode vibration analysis of two dimensional problems including composite laminates, and was reported at the first Japan International Society for the Advancement of Materials and Process Engineering Symposium and Exhibition(JISSE Ⅰ) in 1989[3]. Further extension of this method to the second mode vibration of such two dimensional problems was reported at the Eighth Structures Congress of American Society of Civil Engineers in 1990[4]. This method, applied to thick laminated plates, was reported at The Third East Asia-Pacific Conference on Structural Engineering and Construction (EASEC Ⅲ), 1991, The Eighth International Conference on Composite Materials (ICCM 8), 1991[5], and JISSE Ⅱ, 1991.

The merit of the presented method is that it uses such influence coefficient values, used already for calculating deflection, slope, moment and shear to obtain the natural frequency of the structure. When the plate has concentrated mass or masses, one can simply add these masses to the plate mass and use the same deflection influence surfaces to obtain the natural frequency.

This paper presents the illustration of application of this method to the special orthotropic plates with a pair of opposite edges simply supported and the other pair of opposite edges free. Such plates represent the case of bridge floor systems and decks, and building floors made of advanced composite material panels. This procedure can easily be applied to any type of laminates with arbitrary boundary conditions and non-uniform sections. Several structural elements such as the floor slabs of a factory or a building and others may be subject to point mass/masses in addition to its own masses. Design engineers need to calculate the natural frequencies of such elements but obtaining exact solution to such problems is very much difficult. Pretlove reported a method of analysis of beams with attached masses using the concept of effective mass[12]. This method, however, is useful only for certain simple types of beams. Such problems can be easily solved by presented method. The effect of concentrated mass at the center of the plate on the natural frequency is also presented as an illustration.

In order to illustrate this method, some details already reported by the senior author are repeated in this paper.

## 2. METHOD OF ANALYSIS

A natural frequency of a structure is the frequency under which the deflected mode shape corresponding to this frequency begins to diverge under the resonance condition. From the deflection caused by the free vibration, the force required to make this deflection can be found, and from this force, resulting deflection can be obtained. If the mode shape as determined by the series of this process is sufficiently accurate, then the relative deflections (maximum) of both the converged and the previous one should remain unchanged under the inertia force relate with this natural frequency.

Vibration of a structure is a harmonic motion and the amplitude may contain a part expressed by a trigonometric function. Considering only the first mode as a start, the deflected shape of a structural member can be expressed as

$$w = W(x,y)F(t) = W(x,y)\sin\omega t \tag{1}$$

where

W : the maximum amplitude

$\omega$ : the circular frequency of vibration

t : time.

By Newton's Law, the dynamic force of the vibrating mass, m, is

$$F = m\frac{\partial^2 w}{\partial t^2} \tag{2}$$

Substituting Equation (1) into this,

$$F = -m(\omega)^2 W\sin\omega t \tag{3}$$

In this expression, $\omega$ and W are unknowns. In order to obtain the natural circular frequency, $\omega$, the following process is taken. The magnitudes of the maximum deflection at a certain number of points are arbitrarily given as

$$w(i,j)(1) = W(i,j)(1) \tag{4}$$

where (i,j) denotes the point under consideration. This is absolutely arbitrary but educated guessing is good for accelerating convergence. The dynamic force corresponding to this(maximum) amplitude is

$$F(i,j)(1) = m(i,j)[\omega(i,j)(1)]^2 w(i,j)(1) \tag{5}$$

The "new" deflection caused by this force is a function of F and can be expressed as

$$\begin{aligned} w(i,j)(2) &= f\{m(k,l)[\omega(i,j)(1)]^2\, w(k,l)(1)\} \\ &= \sum^{k,l} \Delta(i,j,k,l)\{m(k,l)[\omega(i,j)(1)]^2 \\ &\quad w(k,l)(1)\} \end{aligned} \tag{6}$$

where $\Delta$ is the deflection influence surface. The relative (maximum) deflections at each point under consideration of a structural member under resonance condition, w(i,j)(1) and w(i,j)(2), have to remain unchanged and the following condition has to be held :

$$w(i,j)(1) / w(i,j)(2) = 1 \tag{7}$$

From this equation, $\omega(i,j)(1)$ at each point of (i,j) can be obtained, but they are not equal in most cases. Since the natural frequency of a structural member has to be equal at all points of the member, i.e., $\omega(i,j)$ should be equal for all (i,j), this step is repeated until sufficient equal magnitude of $\omega(i,j)$ is obtained at all (i,j) points. However, in most cases, the difference between the maximum and the minimum values of $\omega(i,j)$ obtained by the first cycle of calculation is sufficiently negligible for engineering purposes. The accuracy can be improved by simply taking the average of the maximum and the minimum, or by taking the value of $\omega(i,j)$ where the deflection is the maximum. For the second cycle, w(i,j)(2) in

$$w(i,j)(3) = f\{m(i,j)\,[\omega(i,j)(2)]^2 w(i,j)(2)\} \tag{8}$$

the absolute numerics of w(i,j)(2) can be used for convenience.

In case of a structural member with irregular section including composite one, and non-uniformly distributed mass, regardless of the boundary conditions, it is convenient to consider the member as divided by finite number of elements. The accuracy of the result is proportional to the accuracy of the deflection calculation.

## 3. NUMERICAL EXAMPLE

### 3.1 Some Orientations Which Behave as Special Orthotropic Plates

The material properties are assumed as

$E_1 = 38.6$ GPa, $E_2 = 8.27$ GPa,
$\upsilon_{12} = 0.26$, $\upsilon_{21} = 0.0557$,
$G_{12} = 4.14$ GPa, $a = b = 1$m and
$h_o = 0.00125$ m for all plies.

Normalized stiffnesses are defined as

$A^* = A/h$ in GPa
$B^* = 2B/h2$ in GPa
$D^* = 12D/h3$ in GPa

where h and A are laminate thickness and the extensional stiffness matrix, respectively.

$[\alpha\beta\beta\alpha\alpha\beta]_r$ and $[\alpha\beta]_r$ orientations have decreasing values of $B_{16}$, $B_{26}$, $D_{16}$, and $D_{26}$ if $\alpha = -\beta$ and the number of plies, r, increases[1]. $[\alpha\beta\beta\gamma\alpha\alpha\beta]_r$ has the same property if $\gamma$ is either 0° or 90°. For all orientations with above condition $A^*_{11} / D^*_{11} = 1$, indicating that these laminates are quasi-homogeneous.

The result in these tables indicates that the three partial differential equations for the laminate bending,

$$A_{11}\frac{\partial^2 u}{\partial x^2}+2A_{16}\frac{\partial^2 u}{\partial x\partial y}+A_{66}\frac{\partial^2 u}{\partial y^2}+A_{16}\frac{\partial^2 v}{\partial x^2}+(A_{12}+A_{66})\frac{\partial^2 v}{\partial x\partial y}+A_{26}\frac{\partial^2 v}{\partial y^2}-B_{11}\frac{\partial^3 w}{\partial x^3}-3B_{16}\frac{\partial^3 w}{\partial x^2\partial y}-(B_{12}+2B_{66})\frac{\partial^3 w}{\partial x\partial y^2}+B_{26}\frac{\partial^3 w}{\partial y^3}=0 \tag{9a}$$

$$A_{16}\frac{\partial^2 u}{\partial x^2}+(A_{12}+A_{66})\frac{\partial^2 u}{\partial x\partial y}+A_{26}\frac{\partial^2 u}{\partial y^2}+A_{66}\frac{\partial^2 v}{\partial x^2}+2A_{26}\frac{\partial^2 v}{\partial x\partial y}+A_{22}\frac{\partial^2 v}{\partial y^2}-B_{16}\frac{\partial^3 w}{\partial x^3}-(B_{12}+2B_{66})\frac{\partial^3 w}{\partial x^2\partial y}-3B_{26}\frac{\partial^3 w}{\partial x\partial y^2}+B_{22}\frac{\partial^3 w}{\partial y^3}=0 \tag{9b}$$

$$D_{11}\frac{\partial^4 w}{\partial x^4}+4D_{16}\frac{\partial^4 w}{\partial x^3\partial y}+2(D_{12}+2D_{66})\frac{\partial^4 w}{\partial x^2\partial y^2}+4D_{26}\frac{\partial^4 w}{\partial x\partial y^3}+D_{22}\frac{\partial^4 w}{\partial y^4}-B_{11}\frac{\partial^3 u}{\partial x^3}-3B_{16}\frac{\partial^3 u}{\partial x^2\partial y}$$

$$-(B_{12}+2B_{66})\frac{\partial^3 u}{\partial x\partial y^2}-B_{26}\frac{\partial^3 u}{\partial y^3}-B_{16}\frac{\partial^3 v}{\partial x^3}$$

$$-(B_{12}+B_{66})\frac{\partial^3 v}{\partial x^2\partial y}-3B_{26}\frac{\partial^3 v}{\partial x\partial y^2}-B_{22}\frac{\partial^3 v}{\partial y^3}$$

$$= q(x,y) \qquad (9c)$$

can be reduced to one equation, for the special orthotropic plate,

$$D_1\frac{\partial^4 w}{\partial x^4}+2D_3\frac{\partial^4 w}{\partial x^2\partial y^2}+D_2\frac{\partial^4 w}{\partial y^4} = q(x,y) \qquad (10)$$

### 3.2 Vibration of Plates

As a numerical example, the special orthotropic laminate given in Kim's book1) is considered, Fig. 1.

This example illustrates the method of analysis. The material properties are :

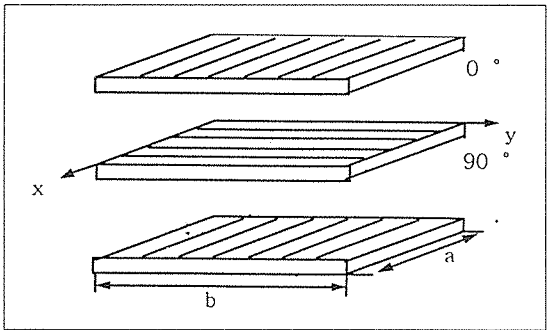
$E_1 = 67.36$ GPa, $E_2 = 8.12$GPa

$G_{12} = 3.0217$GPa

$$\nu_{21} = \nu_{12}\cdot\frac{E_2}{E_1} = 0.0328$$

$$\nu_{12} = V_m\nu_m+V_f\nu_f = 0.2720$$

The stiffnesses are




$t_1 = t_2 = t_3 = 0.005$ m, a = b = 1m
Fiber Orientation : 0°/90°/0°

Fig. 1. Specially Orthotropic Laminate

$$A_{ij}=\sum_{k=1}^{n}(\bar{Q}_{ij})_k\cdot(h_k-h_{k-1}),\ \text{in N/m} \qquad (11)$$

$$B_{ij}=\frac{1}{2}\sum_{k=1}^{n}(\bar{Q}_{ij})_k\cdot(h^2_k-h^2_{k-1}),\ \text{in N} \qquad (12)$$

$$D_{ij}=\frac{1}{3}\sum_{k=1}^{n}(\bar{Q}_{ij})_k\cdot(h^3_k-h^3_{k-1}),\ \text{in Nm} \qquad (13)$$

and obtained as

$$D(i,j)=\begin{vmatrix}18492 & 627 & 0\\ 627 & 2927 & 0\\ 0 & 0 & 849\end{vmatrix}(N-m)$$

$$A(i,j)=\begin{vmatrix}720.67 & 33.43 & 0\\ 33.43 & 421.79 & 0\\ 0 & 0 & 45.33\end{vmatrix}(MN/m)$$

B(i,j) = 0 from symmetry.

The influence surfaces are calculated by

$$w(x,y)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}w_{mn}\cdot\sin\frac{m\pi x}{a}\cdot\sin\frac{n\pi x}{b} \qquad (14)$$

where

$$w_{mn}=\frac{P_{mn}/\pi^4}{D_{11}\left(\frac{m}{a}\right)^4+2(D_{12}+2D_{66})\left(\frac{m}{a}\right)^2\left(\frac{n}{b}\right)^2+D_{22}\left(\frac{n}{b}\right)^4} \qquad (15)$$

in which

$$P_{mn}=\frac{4(1)}{ab}\cdot\sin\frac{m\pi\xi}{a}\cdot\sin\frac{n\pi\eta}{b} \qquad (16)$$

From Eqn (4),

w(i,j)(1) = W(i,j)(1)

where W is the maximum amplitude, (i,j) or (x,y) is the point under consideration, and (1) after (i,j) indicates the first assumed mode shape. The first mode shape is assumed as

$$w(i,j)(1) = \begin{vmatrix} 10 & 20 & 30 & 20 & 10 \\ 20 & 30 & 40 & 30 & 20 \\ 30 & 40 & 50 & 40 & 30 \\ 20 & 30 & 40 & 30 & 20 \\ 10 & 20 & 30 & 20 & 10 \end{vmatrix}$$

By Eqn (5) which is

$$F(i,j)(1) = +m(i,j)\,[\omega(i,j)(1)]^2 w(i,j)(1)$$

where

$$m(i,j) = \text{the mass at } (i,j) \text{ point} = \rho h(i,j)\,\Delta x \Delta y,$$

in which $\Delta$x and $\Delta$y are the mesh sizes in the x- and y- directions, respectively, $\rho$ is the mass density at (i,j), and h is the thickness of the plate at (i,j), and $\omega(i,j)(1)$ is the "first" natural circular frequency at (i,j) point, F(i,j)(1) is obtained in terms of $\omega(i,j)(1)$.

Substituting F(i,j)(1) into Eqn (6)

$$\begin{aligned} w(i,j)(2) &= \sum^{k,l} \Delta(i,j,k,l) \cdot F(i,j)(1) \\ &= \sum^{k,l} \Delta(i,j,k,l) \cdot \{+m(k,l) \cdot \\ &\quad [\omega(i,j)(1)]^2 \cdot w(k,l)(1)\} \end{aligned}$$

where $\Delta$ (i,j,k,l) is the influence surface, i.e., the deflection at (i,j) point caused by a unit load at all of (k,l) points, w(i,j)(2) can be obtained.

From Eqn (7)

$$w(i,j)(1) \,/\, w(i,j)(2) = 1$$

from which, one can obtain

$$\omega(2,2)(1) = 1512.68 \,/\, \sqrt{m(2,2)}$$

The calculation is carried out at all points (i,j) and

$$\omega(i,j)(1) = (1913\text{-}1512) \,/\, \sqrt{m(i,j)}.$$

Since the range of $\omega(i,j)(1)$ is too large, one more cycle is proceeded. For $\omega(i,j)(2)$ to be used for F(i,j)(2), the absolute numerics of $\omega(i,j)(2)$ are used.

Then

$$\omega(i,j)(2) = (1586.5 - 1631.3) \,/\, \sqrt{m(i,j)}$$

$$\omega(2,2)(2) = 1587 \,/\, \sqrt{m(2,2)}.$$

Proceeding further,

$$\omega(i,j)(3) = (1592.5\text{-}1598.0) \,/\, \sqrt{m(i,j)}$$

$$\omega(i,j)(4) = (1593.6\text{-}1594.3) \,/\, \sqrt{m(i,j)}$$

$$\omega(2,2)(3) = 1593 \,/\, \sqrt{m(2,2)}$$

$$\omega(2,2)(4) = 1593.6 \,/\, \sqrt{m(2,2)}.$$

The result by the energy method is

$$\omega = 1593.7 \,/\, \sqrt{m}$$

### 3.3 Application to the Subject Problem

In order to apply the presented method to the subject problem, the first step to take is obtaining the influence surfaces. Any method can be used. Levy solution with sine Fourier series terms to the simply supported edge direction may be good.

Since the "free" edges may have integrally built in "beams", resulting to change of "free" edges to elastically supported edges, the F.E.M. program ARGOL is used for this illustration.

In the ARGOL program, element is used the 4-node sandwich plate element. It is based on Mindlin plate theory. Basic assumptions include :

1) All displacement are small w. r. t. the plate thickness.

2) Each ply of the laminate is linear elastic.

3) Through the thickness stress is assumed zero.

4) The line normal to the surface does not have

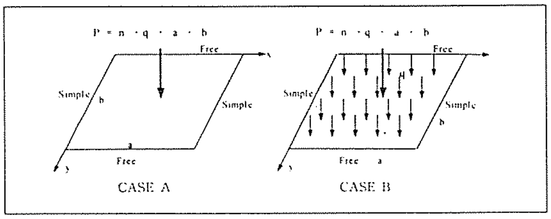



Fig. 2. Plate under Consideration

to remain normal after deformation.

5) Transverse shears are included for the sandwich core only.

6) The face sheets are assumed to obey thin plate theory.

Each node has five degrees of freedom :

1) Two perpendicular in-plane displacements (u, v).

2) One out-of-plane displacement (w).

3) Two out-of-plane rotations ($r_x$, $r_y$).

The accuracy of the natural frequency is proportional to that of the inverse of the square root of the deflection. The accuracy of using this F.E.M. program can be checked by comparing the deflection obtained by this method with that obtained by the other method.

For the plate with four edges simply supported, for which "exact" energy method solution is available, three significant figures by F.E.M, are same as those obtained by the energy method, for almost all values of c=b/a, Fig. 1. The beam analogy has similar result for $c \geq 7$.

For c=4, two figures are equal. For two edges simply supported and the other opposite edges free, the beam analogy and F.E.M. solutions have three significant figures equal when $c \geq 5$.

Table 1. $\omega \cdot \sqrt{\rho h}$, c=a/b=1(a=1m, b=1m), P = N a b q, q =$\rho hg$
CASE A : q Neglected, CASE B : q Considered

| N | CASE A) (rad/sec | CASE B (rad/sec) | $\omega$(B) / $\omega$(A) (%) |
|---|---|---|---|
| 0 | | 1358.161 | |
| 1 | 763.3866 | 686.2808 | 89.89 |
| 3 | 440.7415 | 424.8031 | 96.38 |
| 5 | 341.3969 | 333.8809 | 97.79 |
| 7 | 288.5330 | 283.9673 | 98.42 |
| 10 | 241.4040 | 238.7175 | 98.89 |
| 15 | 197.1056 | 195.6378 | 99.26 |
| 20 | 170.6984 | 169.7433 | 99.44 |

For the plate with all edges fixed, the beam analogy solution has differences less than 0.6% when $c \geq 5$.

For this illustration, a concentrated load P=N a b q at the center of the plate is added to the uniform load q. The influence of the concentrated attached mass is studied by increasing N for two cases, namely, CASE A=q neglected, CASE b=q considered. where N is real number. The result is given in Table 1. Figure 3 is Natural frequency for each cases. Fig. 4. is Natural frequency ratio.

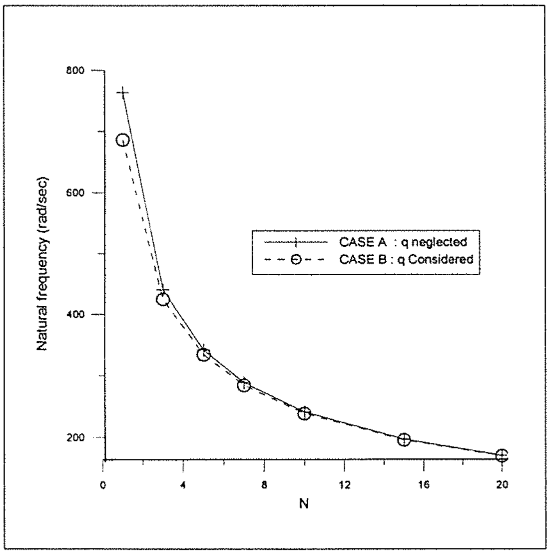



Fig. 3. Natural frequency for each cases

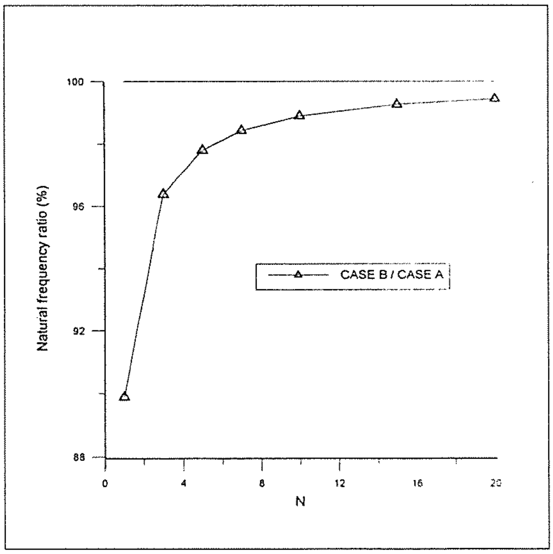



Fig. 4. Natural frequency ratio (%)

## 4. CONCLUSION

In this paper, the simple and accurate method of vibration analysis developed by D. H. Kim is presented with detailed illustration.

Numerical illustrations for beams and plates are given and it is proven that the presented method is simple to use but extremely accurate. The boundary condition can be arbitrary. Both stiffness and mass of the element can be variable. One can use any method to obtain the deflection influence coefficient. The accuracy of the solution is dependent on only that of the influence coefficients. One should recall that obtaining the deflection influence coefficients is the first step in design and analysis of a structure. The merit of the presented method is that it uses such influence coefficient values, used already for calculating deflection, to obtain the natural frequency of the structure. When the plate has concentrated mass or masses, one can simply add these masses to the plate mass and use the same deflection influence surfaces to obtain the natural frequency.

This method is applied to the special orthotropic plate with two opposite edges simply supported and with the other opposite two edges free, Fig.1, 2. Such plate is the case of the most of the simply supported bridges.

Several laminates with certain orientations reduce the three Partial differential equations to one equation for the special orthotropic plate. When advanced composite materials are used for bridges, buildings, and other civil constructions, the design must have fiber orientations other than $0^{\circ}$ and $90^{\circ}$. Analysis of such laminate is very much complex because of the existence of three simultaneous Partial differential equations. The laminates given in this paper have the advantages of such fiber orientations but can be analysed by simple method used for the special orthotropic orientations. Further simple method can be used for such laminates. Such method is given in Reference[1].

The presented method can be applied to such laminates as well as concrete bridge/decks, and the girders and cross-beams systems.

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