

論文

탄성구속된 직교이방성판의 좌굴거동

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Buckling Behavior of Elastically Restrained Orthotropic Plates

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초 록

본 연구는 탄성구속된 직교이방성판의 탄성좌굴거동에 대한 해석적인 연구결과이다. 이 연구에서 판의 하중이 재하되는 변은 단순지지되어있고 하중이 재하되지 않는 변은 탄성구속되어있다. 등분포 일축 압축력과 면내 전단력이 작용하는 경우에 대한 좌굴해석식을 에너지법을 사용하여 각각 유도하였으며 그 결과를 그래프로 나타내었다. 유도된 식의 검증을 위하여 등방성재료의 역학적성질을 좌굴해석식에 대입한 결과 기존의 발표된 등방성재료의 결과와 동일한 결과를 얻을 수 있었다.

ABSTRACT

In this paper, we present the analytical study results of buckling behavior of elastically restrained orthotropic plates. In the study the boundary conditions of the plate are simply supported at all four edges and elastically restrained by the elastic medium at opposite two longitudinal edges. The energy method is employed in the solution of the problems in which method the buckling coefficient is calculated from the condition that the work-done by the external forces during buckling is equal to the stored elastic strain energy. The results are presented in the graphical form. The equations derived for the orthotropic plate in this study are compared with existing isotropic ones and identical results were observed.

INTRODUCTION

Due to the rapid developments and advances in the field of manufacturing composites, fiber reinforced polymeric plastic composite structural shapes are produced using the pultrusion process which is known to be one of the most cost effective manufacturing techniques. Considering the manufacturing process of the pultruded materials,

this material is frequently assumed to be orthotropic or more specifically transversely isotropic.

The fiber reinforced plastic composite structural shapes are readily available in the field of civil engineering. These composite structural shapes are used in the construction of new structures, and rehabilitation and strengthening of deteriorated existing structures under service. Such a trend is expected to continue because the material is rec-

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ognized as alternatives to overcome the deficiency arising in the use of the conventional construction materials such as steel and/or concrete. The fiber reinforced polymeric composite material has an attractive mechanical and physical properties such as light weight, high strength to weight ratio, non-conductivity, nonmagnetic properties, high corrosion resistance, and so on. In spite of the superior mechanical and physical properties, many engineers are reluctant to design with fiber reinforced plastic structural members because of the lack of reliable design criteria.

In general, the structural shapes such as I-, channel, and box sections are composed of flat plate elements. When such a member is used as a column or a beam, the member must be strong to resist external loads such as compression, bending, and shear.

One of the most important failure modes of the structural member is the local buckling. In the local buckling modes, plate components buckle before the global (or Euler) buckling or material yielding take place. Therefore, in order to establish the design criteria, thorough research on the buckling behavior of plate under various loading and boundary conditions must be conducted.

In this paper we present the analytical results of an elastic buckling behavior of orthotropic plate under uniform compression and in-plane shear, respectively. In the analysis, opposite two edges are assumed to be equally elastically restrained while other two opposite edges are simply supported. In addition, this study is limited to the elastic buckling behavior, because most of the plastic structural shapes used in the civil engineering construction are composed of brittle materials and the post-buckling strength is relatively small compared with elastic buckling strength.

The study of elastically restrained isotropic plates have been made by many researchers such as Timoshenko[1], Nölke[2], Schuette, and McCulloch[3]. But the study on the buckling of an orthotropic plate having elastically restrained

edges is not available.

The objective of this research is to derive an analytical equations for finding the buckling strength of simply supported orthotropic plate whose two opposite boundaries are additionally elastically restrained, and it is subjected to uniform axial compression and in-plane shear force, respectively.

BASIC EQUATIONS FOR THE ORTHOTROPIC PLATE BUCKLING

Due to the complexity of the problem, we adopt the energy method suggested by Timoshenko. The plate was considered to be simply supported along the two opposite short (loaded) edges and elastically restrained by a restraining medium along the two opposite long (unloaded) edges. It was assumed that the restraining medium was such that a sinusoidally applied moment causes a sinusoidal rotation at unloaded edges in the plate. This condition is satisfied by a rigid joint between two or more flat plates that buckle into the same wave length (Johnson and Noel, 1953)[4]. Material property directions, coordinate systems, and the external loads subjecting to the plates are shown in Fig. 1 and Fig. 2.

In accordance with the classical theory of orthotropic plates (Lekhnitskii, 1984)[5], the strain energy of orthotropic rectangular thin plates is given by:

$$U = \frac{1}{2} \int_{-\frac{a}{2}}^{\frac{a}{2}} \int_{-\frac{b}{2}}^{\frac{b}{2}} \left[D_{11} \left(\frac{\partial^2 w}{\partial x^2} \right)^2 + 2D_{11} v_{21} \left(\frac{\partial^2 w}{\partial x^2} \right) \cdot \left(\frac{\partial^2 w}{\partial y^2} \right) + D_{22} \left(\frac{\partial^2 w}{\partial y^2} \right)^2 + 4D_{66} \left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 \right] dx dy \quad (1)$$

The work done by the uniform compression (V_c) and in-plane shear (V_s) forces is expressed by, respectively:

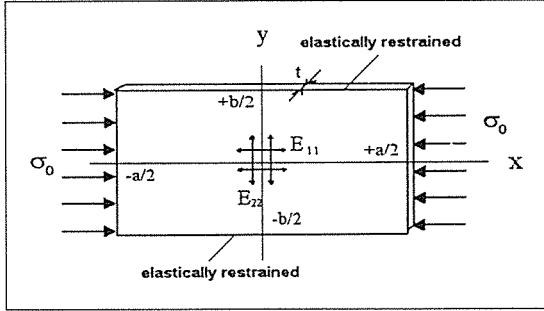


Fig. 1. Orthotropic plate under uniform compression

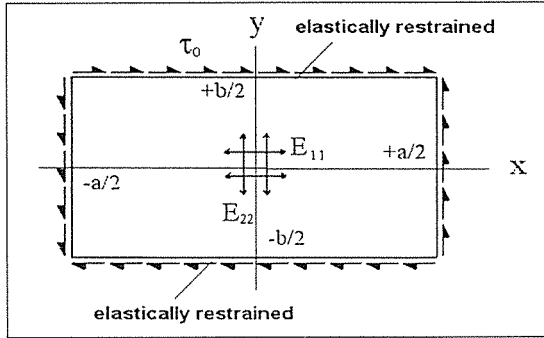


Fig. 2. Orthotropic plate under in-plane shear

$$V_c = -\frac{\sigma_0 t}{2} \int_{-\frac{a}{2}}^{\frac{a}{2}} \int_{-\frac{b}{2}}^{\frac{b}{2}} \left(\frac{\partial w}{\partial x} \right)^2 dx dy \quad (2)$$

$$V_s = \tau_0 t \int_{-\frac{a}{2}}^{\frac{a}{2}} \int_{-\frac{b}{2}}^{\frac{b}{2}} \left(\frac{\partial w}{\partial x} \right) \left(\frac{\partial w}{\partial y} \right) dx dy \quad (3)$$

In Eqs. (2) and (3), $\tau_0 t$ and $\sigma_0 t$ are in-plane shear and uniform compression forces, respectively. Plate length and width are a and b , and plate thickness t is assumed to be uniform throughout the plate. In Eqs. (1) to (3) w is the out of plane deflection due to in-plane shear or uniform compression forces. In Eq. (1) D_{11} and D_{22} are the flexural rigidities of the plate with respect to the material property directions 1 and 2, D_{66} is the twisting rigidity of the plate, and ν_{12} and ν_{21} are the major and minor Poisson's ratios.

These are defined as follows:

$$D_{11} = \frac{E_{11} t^3}{12(1 - \nu_{12}\nu_{21})}, \quad D_{22} = \frac{E_{22} t^3}{12(1 - \nu_{12}\nu_{21})}$$

$$D_{66} = \frac{G_{12} t^3}{12}, \quad \nu_{12} = \nu_{21} \frac{E_{11}}{E_{22}} \quad (4a,b,c,d)$$

The expression for strain energy in the two equal elastic restraining mediums, U_r , is (Bulson, 1969)[6]:

$$U_r = \frac{M_r}{2} \left[\int_{-\frac{a}{2}}^{\frac{a}{2}} \left\{ \left(\frac{\partial w}{\partial y} \right)_{y=-\frac{b}{2}} \right\}^2 dx + \int_{-\frac{a}{2}}^{\frac{a}{2}} \left\{ \left(\frac{\partial w}{\partial y} \right)_{y=+\frac{b}{2}} \right\}^2 dx \right] \quad (5)$$

In Eq. (5) M_r is the moment per unit length applied by the restraining medium per unit rotation. If the torsional rigidity of restraining plate is C_L , the moment M_r can be expressed in terms of C_L as follows (Bulson, 1969)[6]:

$$M_r = -\frac{\pi^2 C_L}{a^2} \quad (6)$$

Using the coordinate system shown in Fig. 1 and Fig. 2, the assumed deflection surface can be taken following Lundquist and Stowell(1942)[7]:

$$w = B \left[\frac{\pi \varepsilon}{2b^2} \left(y^2 - \frac{b^2}{4} \right) + \left(1 + \frac{\varepsilon}{2} \right) \cdot \cos \frac{\pi y}{b} \right] \cos \frac{m \pi x}{a} \quad (7)$$

where B is an arbitrary deflection amplitude and m is the number of half sine waves.

In Eq. (7) the symbol ε is a coefficient of restraint or coefficient of fixity which is a dimensionless parameter given by the equation:

$$\varepsilon = \frac{M_r \cdot b}{D_{22}} \quad (8)$$

The restraining coefficient ε depends upon the relative stiffness of the plate and the restraining element along the side edge of the plate (Lundquist and Stowell, 1942)[7].

When $\varepsilon=0$ the edge is hinged, when $\varepsilon=\infty$ it is built-in. As noticed, the coefficient of fixity ε is the ratio of edge moment to edge slope.

So far, we discussed the basic equations for the elastic buckling of rectangular orthotropic plates under uniform compression and in-plane shear, respectively. Using the equations discussed we solve the two different orthotropic plate buckling problems, both equally restrained at the two opposite long edges.

BUCKLING ANALYSIS OF PLATE UNDER UNIFORM COMPRESSION

Substituting Eq. (7) into Eqs. (1) and (3), we can obtain following form of stain energy and work-done by uniform compression for orthotropic plate, respectively:

$$U = \frac{\pi^4 B^2}{4a^2} \left[\frac{m^4}{\phi} \left\{ \left(\frac{\pi^2}{120} - \frac{2}{\pi^2} + \frac{1}{8} \right) \cdot \varepsilon^2 + \left(\frac{1}{2} - \frac{4}{\pi^2} \right) \cdot \varepsilon + \frac{1}{2} \right\} \cdot D_{11} + 2m^2 \phi \left\{ \left(\frac{5}{24} - \frac{2}{\pi^2} \right) \cdot \varepsilon^2 + \frac{1}{2} \right\} \cdot (D_{11} \cdot \nu_{21} + 2D_{66}) + \phi^3 \left\{ \left(\frac{1}{8} - \frac{1}{\pi^2} \right) \cdot \varepsilon^2 + \left(\frac{1}{2} - \frac{4}{\pi^2} \right) \cdot \varepsilon + \frac{1}{2} \right\} \cdot D_{22} \right] \quad (9)$$

$$V_c = \frac{m^2 \pi^2 b}{4a} B^2 \sigma_0 t \cdot \left[\left(\frac{\pi^2}{120} - \frac{2}{\pi^2} + \frac{1}{8} \right) \cdot \varepsilon^2 + \left(\frac{1}{2} - \frac{4}{\pi^2} \right) \cdot \varepsilon + \frac{1}{2} \right] \quad (10)$$

In Eq. (10) σ_0 is expressed by using Timoshenko's expression as follows:

$$\sigma_0 = k_c \frac{\pi^2 \sqrt{E_{11} E_{22}}}{12(1 - \nu_{12} \nu_{21}) \left(\frac{b}{t} \right)^2} \quad (11)$$

where k_c is the buckling coefficient of plate.

In addition, the strain energy in two equal elastic medium can be taken in the form:

$$U_r = \frac{a}{2} \cdot M_r \cdot \frac{B^2 \pi^2}{b^2} \quad (12)$$

Therefore the buckling coefficient of orthotropic plate under uniform compression, by using the relation $V = U + U_r$, can be obtained as:

$$k_c = \frac{m^2}{\phi^2} \left\{ \sqrt{\frac{E_{11}}{E_{22}}} + C_1 \frac{\phi^2}{m^2} \sqrt{\frac{E_{22}}{E_{11}}} + 2C_2 \left(\nu_{21} \sqrt{\frac{E_{11}}{E_{22}}} + \frac{2G_{12}(1 - \nu_{12} \cdot \nu_{21})}{\sqrt{E_{11} \cdot E_{22}}} \right) \right\} \quad (13)$$

where ϕ is the plate aspect ratio (a/b), C_1 and C_2 are defined, respectively:

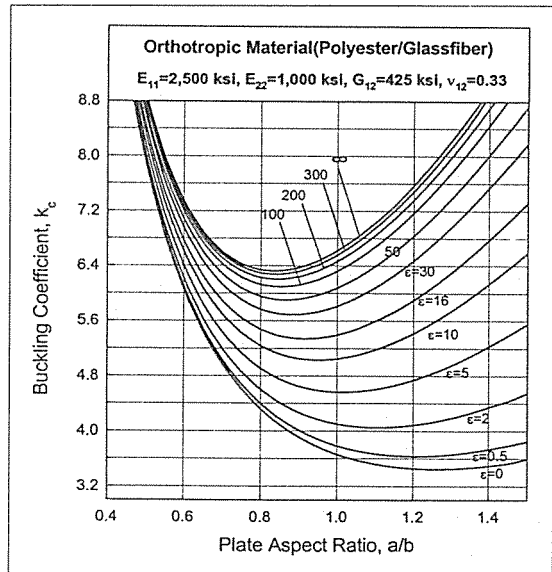


Fig. 3. Buckling coefficient of orthotropic plate under uniform compression

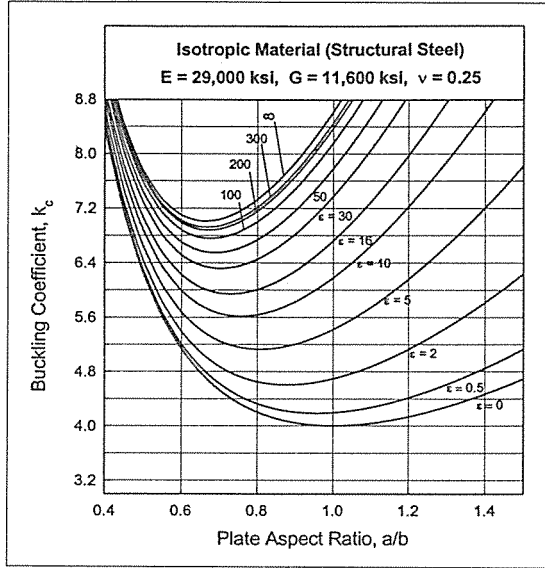


Fig. 4. Buckling coefficient of isotropic plate under uniform compression

$$C_1 = \frac{\left(\frac{1}{8} - \frac{1}{\pi^2}\right)\epsilon^2 + \left(\frac{1}{2} - \frac{2}{\pi^2}\right)\epsilon + \frac{1}{2}}{\left(\frac{\pi^2}{120} - \frac{2}{\pi^2} + \frac{1}{8}\right)\epsilon^2 + \left(\frac{1}{2} - \frac{4}{\pi^2}\right)\epsilon + \frac{1}{2}} \quad (14)$$

$$C_2 = \frac{\left(\frac{1}{8} - \frac{1}{\pi^2}\right)\epsilon^2 + \left(\frac{1}{2} - \frac{4}{\pi^2}\right)\epsilon + \frac{1}{2}}{\left(\frac{\pi^2}{120} - \frac{2}{\pi^2} + \frac{1}{8}\right)\epsilon^2 + \left(\frac{1}{2} - \frac{4}{\pi^2}\right)\epsilon + \frac{1}{2}} \quad (15)$$

Thus we can find the buckling coefficient for elastically restrained orthotropic plate subjected to uniform compression force by calculating Eq. (13) numerically with respect to various coefficients of fixity from 0 to ∞ . The buckling coefficient curves for orthotropic and isotropic plates with various ϵ are drawn in Fig. 3 and Fig. 4, respectively. The orthotropic material properties used to draw Fig. 3 are taken for the structural members in MMFG (1989)[8] which is widely used in many countries. The buckling coefficient with respect to the various values of coefficient of fixity is also drawn and shown in Fig. 5.

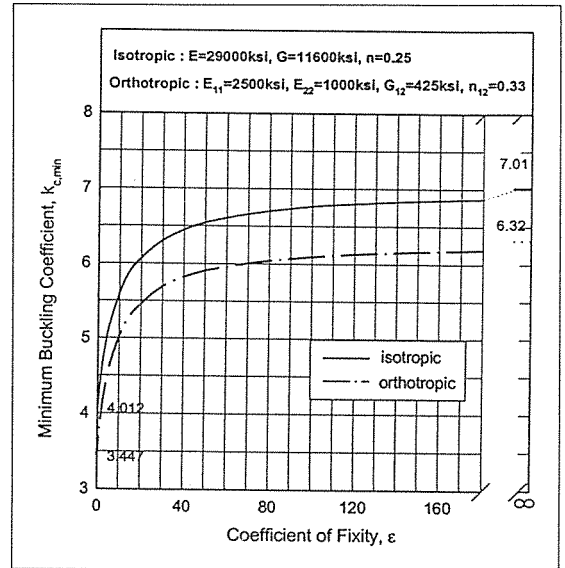


Fig. 5. Minimum buckling coefficient for various coefficients of fixity (uniform compression force)

BUCKLING ANALYSIS OF PLATE UNDER IN-PLANE SHEAR

When buckling occurs in a plate under in-plane shear force, nodal line is not parallel with the axis direction of members. Therefore in energy solution, oblique coordinate system has to be used because under shear buckling conditions the nodal lines are inclined at an angle to the side of the plate (Bulson, 1969)[6].

As shown in Fig. 6, the oblique coordinates (x , y) are related to the cartesian coordinates (x' , y') through the equations as follows:

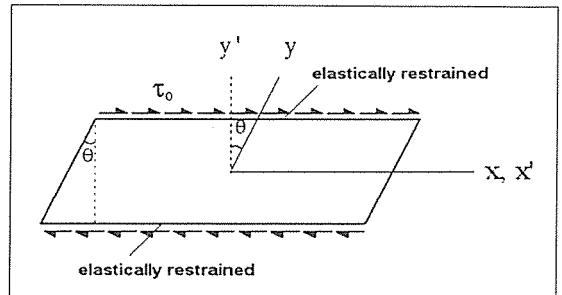


Fig. 6. Oblique coordinate system

$$\begin{aligned}x' &= x - y \sin \theta \\ y' &= y \cos \theta\end{aligned}\quad (16a,b)$$

where θ is the angle of inclination of the nodal lines. Substituting Eq. (16a,b) into Eq. (1), and using the chain rule, the strain energy can be expressed in the form:

$$\begin{aligned}U &= \frac{1}{2} \int_{-\frac{a}{2}}^{\frac{a}{2}} \int_{-\frac{b}{2}}^{\frac{b}{2}} \left\{ \left(\frac{\partial^2 w}{\partial x^2} \right)^2 (D_{11} + 2D_{11}v_{21}\tan^2\theta + D_{22}\tan^2\theta + 4D_{66}\tan^2\theta) \right. \\ &\quad + \left(\frac{\partial^2 w}{\partial y^2} \right)^2 \frac{D_{22}}{\cos^4\theta} + 4 \left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 \left(D_{22} \frac{\tan^2\theta}{\cos^3\theta} + D_{66} \frac{1}{\cos^3\theta} \right) \\ &\quad + 4 \left(\frac{\partial^2 w}{\partial x^2} \right) \left(\frac{\partial^2 w}{\partial x \partial y} \right) D_{22} \frac{\tan\theta}{\cos^3\theta} + 2 \left(\frac{\partial^2 w}{\partial x^2} \right) \left(\frac{\partial^2 w}{\partial y^2} \right) \\ &\quad \left(D_{11}v_{21} \frac{\tan\theta}{\cos\theta} + 2D_{22} \frac{\tan^3\theta}{\cos\theta} + 4D_{66} \frac{\tan\theta}{\cos\theta} \right) + 2 \left(\frac{\partial^2 w}{\partial x^2} \right) \left(\frac{\partial^2 w}{\partial y^2} \right) \\ &\quad \cdot \left(D_{11}v_{21} \frac{1}{\cos^3\theta} + D_{22} \frac{\tan^2\theta}{\cos^3\theta} \right) \Big\} dx dy\end{aligned}\quad (17)$$

The work done by the in-plane shear force is

$$V_s = \tau_0 t \int_{-\frac{a}{2}}^{\frac{a}{2}} \int_{-\frac{b}{2}}^{\frac{b}{2}} \left\{ \left(\frac{\partial w}{\partial x} \right) \left(\frac{\partial w}{\partial y} \right) + \left(\frac{\partial w}{\partial x} \right)^2 \sin\theta \right\} dx dy\quad (18)$$

The equation for the strain energy in the two elastic restraining mediums becomes as follows:

$$\begin{aligned}U_r &= \frac{M_r}{2\cos^2\theta} \left[\int_{-\frac{a}{2}}^{\frac{a}{2}} \left\{ \frac{\partial w}{\partial y} + \frac{\partial w}{\partial x} \sin\theta \right\}_{y=-\frac{b}{2}}^2 dx \right. \\ &\quad \left. + \int_{-\frac{a}{2}}^{\frac{a}{2}} \left\{ \frac{\partial w}{\partial y} + \frac{\partial w}{\partial x} \sin\theta \right\}_{y=\frac{b}{2}}^2 dx \right]\end{aligned}\quad (19)$$

The equation representing the deflection surface can also be transformed using oblique coordinates:

$$w = B \left[\frac{\pi \varepsilon}{2b_1^2} \left(y^2 - \frac{b_1^2}{4} \right) + \left(1 + \frac{\varepsilon}{2} \right) \cdot \cos \frac{\pi y}{b_1} \right] \cos \frac{m\pi x}{a}\quad (20)$$

where $b_1 = b/\cos\theta$.

When using the same relation with preceeding paragraph $V = U + U_r$, we can calculate the buckling coefficient of orthotropic plate subjected to in-plane shear force by substituting Eq. (20) into Eqs. (17), (18), and (19) as follows:

$$\begin{aligned}k_s &= -\frac{1}{\sin 2\theta} \left[\frac{m^2}{\phi^2} \left\{ \sqrt{\frac{E_{11}}{E_{22}}} \cos^3\theta + 2 \left(v_{21} \sqrt{\frac{E_{11}}{E_{22}}} + \frac{2G_{12}(1-v_{12} \cdot v_{21})}{\sqrt{E_{11} \cdot E_{22}}} \right. \right. \right. \\ &\quad \left. \left. + \sqrt{\frac{E_{22}}{E_{11}}} \tan^2\theta \cos^3\theta \right) + C_1 \frac{\phi^2}{m^2} \sqrt{\frac{E_{22}}{E_{11}}} \cos^3\theta \right. \\ &\quad \left. \left. + 2C_2 \left(3 \sqrt{\frac{E_{11}}{E_{22}}} \sin^3\theta + Q \cos^3\theta \right) \right\} \right]\end{aligned}\quad (21)$$

where C_1 and C_2 are same with Eq. (13) and (14), respectively. The angle of inclination θ will adjust itself for given values of ϕ and ε to make the buckling coefficient a minimum. This can be achieved by extremization of k_s with respect to θ and by equating the ensuing equation to zero.

$$\begin{aligned}\frac{\partial k_s}{\partial \theta} &= \frac{2\cos\theta}{(\sin\theta)^2} - \frac{1}{\sin 2\theta} \left[\frac{m^2}{\phi^2} \left\{ -2 \sqrt{\frac{E_{11}}{E_{22}}} \cdot \sin\theta \cos\theta \right. \right. \\ &\quad \left. \left. + 4Q \cdot \sin\theta \cos\theta + 2 \sqrt{\frac{E_{22}}{E_{11}}} \cdot \tan^2\theta \cdot (\tan\theta + \sin\theta \cos\theta) \right\} \right. \\ &\quad \left. - 2C_1 \frac{\phi^2}{m^2} \cdot \sqrt{\frac{E_{22}}{E_{11}}} \cos\theta \sin\theta \right. \\ &\quad \left. + 4C_2 \left(3 \sqrt{\frac{E_{22}}{E_{11}}} \sin\theta \cos\theta - Q \cdot \sin\theta \cos\theta \right) \right] = 0\end{aligned}\quad (22)$$

In the above equation, we can find θ which makes k_s a minimum. Then substituting θ into Eq. (21) we may calculate numerically the buckling coefficient of orthotropic plate subjected to in-plane shear force for the plate aspect ratio and various values of coefficient of fixity ε . The graphical form of results is shown in Fig. 6 for orthotropic plates. The graph has been drawn using the same material properties mentioned ear-

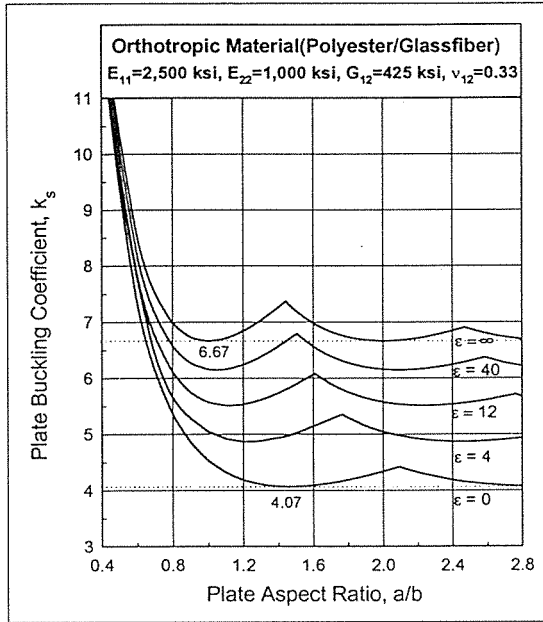


Fig. 7. Buckling coefficient of orthotropic plate subjected to in-plane shear force

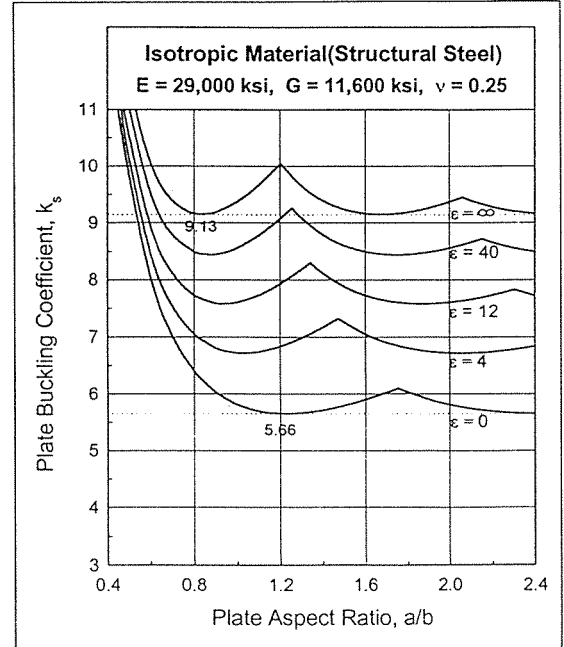


Fig. 8. Buckling coefficient of isotropic plate subjected to in-plane shear force

lier. To verify the accuracy of result orthotropic material properties are substituted with isotropic ones, and the result is shown in Fig. 7. Identical results given in Bleich(1952)[9], Timoshenko (1961)[1], Bulson(1969)[6], etc. are obtained. The graph for the minimum plate buckling coefficient k_s at various coefficients of fixity ϵ is also drawn and presented in Fig. 8 so that the minimum buckling coefficient by in-plane shear force for the various values of coefficient of fixity ϵ can be obtained directly in the figure.

DISCUSSION OF RESULTS AND CONCLUSION

In this analytical study, the elastic buckling behavior of orthotropic plates subjected to uniform compression and in-plane shear were investigated, respectively. The boundary condition of two longitudinal edges was assumed to be equally elastically restrained by the elastic medium. The

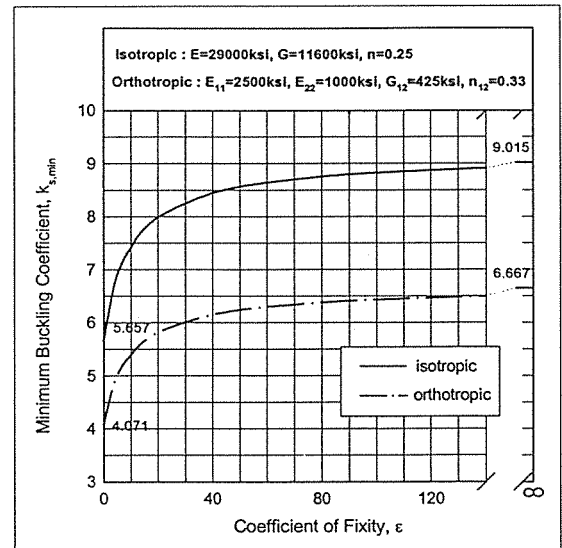


Fig. 9. In-plane shear buckling coefficient for various coefficient of fixity

energy method is employed in the solution of the problems. In order to prove the correctness of derived equations, orthotropic material properties

are substituted with isotropic ones. Then the equations were compared with existing equations given in the literature cited earlier. The identical results were obtained. From both cases of buckling analysis (under uniaxial compression and shear force) when $e=0$, we can obtain the identical result with that of simply supported plate and when $e=\infty$, the result of fixed plate is obtained. Also we can find the buckling coefficient of web plate at various values of e , coefficient of fixity, which is determined according to the dimensions of flange.

Using derived equations, the graphical form of results was suggested so that the elastic buckling strength of elastically restrained orthotropic plates under in-plane shear and uniform compression forces could be calculated easily.

In order to derive complete set of design criteria relating to the local buckling behavior of structural shapes composed of flat plate elements, other loading conditions must also be investigated including linearly varying edge loading. In addition, the investigation is limited to analytic, the experimental investigation must also be conducted to clarify the assumptions lying behind the theory applied.

ACKNOWLEDGEMENTS

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REFERENCES

1. Timoshenko, S. P. and Gere, J. M. Theory of elastic stability, 2nd ed., McGraw-Hill Book Co., Inc., New York, 1961.
2. Nölke, K. "Biegungsbeulung der rechteckplatte mit eingepannten längsrädn," Der Bauingenieur, Vol. 17, pp. 111 and Ingenieur-Archiv, Vol. 8, 1937, pp. 403~425.
3. Schette, E. H. and McCulloch, J. C. Chart for the minimum-weight design of multiweb wings in bending, NACA Technical Note 1323, 1947.
4. Johnson, J. H. and Noel, R. G. "Critical bending stress for flat rectangular plates supported along all edges and elastically restrained against rotation along the unloaded compression edges," Journal Aero. Sci. Vol. 20, No. 8, 1953, pp. 535~540.
5. Lekhnitskii, S. G. Anisotropic plates, Tsai, S. W. and Cheron, T. (Trans.), Gordon and Breach, 2nd printing, New York, 1984.
6. Bulson, P. S. The stability of flat plates, American Elsevier Publishing Company Inc., New York, 1969.
7. Lundquist, E. and Stowell, E. Z., Critical compressive stress for flat rectangular plates supported along all edges and elastically restrained against rotation along the unloaded edges, NACA Technical Report No. 733, 1942.
8. MMFG Extern fiberglass structural shapes design manual, Morrison Molded Fiberglass Company, Bristol, Virginia. 1989.
9. Bleich, F. Buckling strength of metal structures, McGraw-Hill Book Co., Inc., New York, 1952.
10. Jung, J. H., Jeong, S. K., Chae, S. H., and Yoon, S. J. "Shear buckling of long orthotropic thin plate with two elastically restrained edges," Proceedings of the Korean Society for Composite Materials Conference, Vol. 1, 1998, pp. 167~170.
11. Yoon, S. J., Park, B. H., and Jeong, S. K. "Stability of simply supported orthotropic rectangular plates under in-plane shear and bending force," Journal of the Korean Society for Composite Materials, Vol. 11, 1998, No.6, pp. 10~20.
12. Stein, M. and Neff, J. Buckling stresses of simply supported rectangular flat plates in shear, NACA Technical Note No. 1222, National Advisory Committee for Aeronautics, Langley Memorial Aeronautical Laboratory, Langley Field, Virginia, 1947.

UNIT CONVERSION

$$1\text{ksi}=1\text{kip/in}^2=70.31\text{kg/cm}^2$$

$$1\text{kip}=1000\text{lb}=453.6\text{kg},$$

$$1\text{in}=2.54\text{cm}.$$