論文

면내 전단력과 휨을 동시에 받는 직교이방성판의 탄성좌굴강도

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Elastic Buckling Strength of Orthotropic Plate under Combined In-plane Shear and Bending Forces.

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초 록

본 연구는 면내 전단과 휨을 동시에 받는 직교이방성 구조재의 복부판의 탄성좌굴거동에 대한 해석적 연구결과이다. 펄트루젼 방법으로 생산된 섬유보강 플래스틱을 직교이방성으로 간주하였다. 복부판의 좌굴해석에서 직교이방성 재료의 역학적 성질을 고려할 수 있도록 기존의 등방성 판의 좌굴해석을 위해 개발된 이론적 해를 확장하였다. 이론식의 해를 구하는 방법으로 Rayleigh-Ritz법을 사용하였으며, 면내 전단과 휨을 동시에 받는 복부판의 탄성좌굴강도를 구할 수 있는 그래프를 제시하였다. 전단과 휨의 상호작용과 관련된 설계기준에 대해서도 또한 검토하였다.

ABSTRACT

In this paper result of an analytical investigation pertaining to the elastic buckling behavior of orthotropic plate under combined in-plane shear and bending forces is presented. The existing analytical solution developed for the isotropic plates is extended so that the orthotropic material properties can be taken into account in the buckling analysis of web plate. For the solution of the problems Rayleigh-Ritz method is employed. Graphical form of results for finding the elastic buckling strength of orthotropic plate under combined in-plane shear and bending forces is presented. Brief discussion on the design criteria for the shear and bending interaction is also presented.

1. Introduction

In this paper, we presented the analytical results of buckling of orthotropic plate under combined in-plane shear and bending forces.

In relation to the problems discussed herein, Yoon et al.(1998)[1] presented the analytical solution for the elastic buckling strength of web plate under in-plane shear and bending forces acting separately. In the analysis, boundary conditions of plate was assumed to be simply supported and Rayleigh-Ritz method was adopted.

In general, in-plane shear and bending forces are usually acting simultaneously in the web of beams and/or girders. Therefore, in the buckling analysis of web of flexural members, not only the

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simultaneous action of in-plane shear and bending forces but also the interaction between those forces should be considered.

2. Theoretical Derivations

In this paper we solved the simply supported orthotropic rectangular plates under simultaneously acting in-plane shear and bending forces as shown in Fig. 1 and Fig. 2.

by employing the Rayleigh-Ritz method. The method adopted in this study was originally applied by Way(1936)[2] to the isotropic plates. By the application of this method desired accuracy can be achieved by simple expansion of the series with more terms.

Details of derivation were presented by Yoon et al.(1998)[1]. Therefore, we explain the differences of derivation procedure between the present study and results published already by authors.

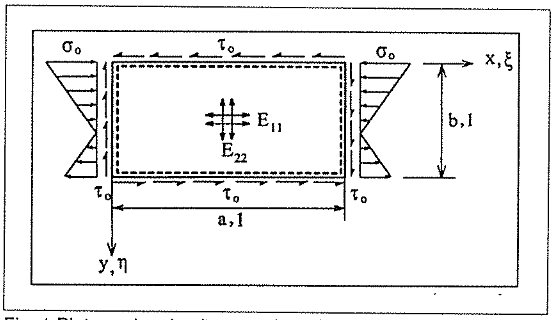

Fig. 1 Plate under simultaneously acting in-planeshear and bending stresses

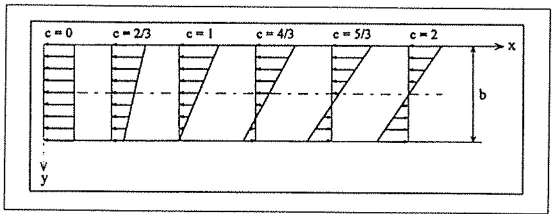

Fig. 2 Numerical factor c for bending stresses

The deflection w of the simply supported plate can be taken in the form of double trigonometric sine-series following Navier' s approach(Lekhnitskii, 1984)[3].

$$w=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}A_{mn}\sin m\pi\xi \sin n\pi\eta \qquad (1)$$

The total potential energy, Π, of orthotropic plate due to combined in-plane shear and bending forces is given by:

$$\Pi = U-(V_b+V_s) \qquad (2)$$

In above equation, U is strain energy of orthotropic plate under combined loading, V_b is work-done by linearly distributed load, and V_s is work-done by in-plane shear force, respectively. Each term in Eq . (2) can be defined with nondimensionalized form as follows, respectively:

$$U=\frac{ab}{2}\int_0^1\int_0^1\left\{\frac{D_{11}}{a^4}\left(\frac{\partial^2 w}{\partial\xi^2}\right)^2 + \frac{2D_{11}\nu_{21}}{a^2b^2}\left(\frac{\partial^2 w}{\partial\xi^2}\right)\left(\frac{\partial^2 w}{\partial\eta^2}\right) + \frac{D_{22}}{b^4}\left(\frac{\partial^2 w}{\partial\eta^2}\right)^2 + \frac{4D_{66}}{a^2b^2}\left(\frac{\partial^2 w}{\partial\xi\partial\eta}\right)^2\right\}d\xi d\eta \qquad (3)$$

$$V_b=\frac{ab}{2}\sigma_0 t\int_0^1\int_0^1\frac{1}{a^2}(1-c\eta)\left(\frac{\partial w}{\partial\xi}\right)^2 d\xi d\eta \qquad (4)$$

$$V_s=-\tau_0 t\int_0^1\int_0^1\left(\frac{\partial w}{\partial\xi}\right)\left(\frac{\partial w}{\partial\eta}\right)d\xi d\eta \qquad (5)$$

In Eqs. (4) and (5), the in-plane shear and bending stresses may be expressed with buckling coefficients k_s and k_b defined as below following Timoshenko' s approach:

$$\tau_{cr}=\frac{k_s\pi^2\sqrt{D_{11}D_{22}}}{b^2 t}=\frac{k_s\pi^2\sqrt{E_{11}E_{22}}}{12(1-\nu_{12}\nu_{21})\left(\frac{b}{t}\right)^2} \qquad (6)$$

$$\sigma_{cr} = \frac{k_b \pi^2 \sqrt{D_{11} D_{22}}}{b^2 t} = \frac{k_b \pi^2 \sqrt{E_{11} E_{22}}}{12(1-\nu_{12}\nu_{21})\left(\frac{b}{t}\right)^2} \qquad (7)$$

The coefficient A_{mn} must be chosen to make the value of k_s and/or k_b a minimum. Using the Rayleigh-Ritz method, the minimization of k_s and/or k_b with respect to each Amn results in the set of homogeneous linear equations represented by the Eq. (8). In this equation, ψ=a/b, m = 1, 2, 3, ⋯, n = 1, 2, 3, ⋯, and m+p and n+q are always odd numbers.

$$\frac{\pi^2}{32\psi^3}\left\{ m^4 \sqrt{\frac{E_{11}}{E_{22}}} + 2\nu_{21}(mn\psi)^2 \sqrt{\frac{E_{11}}{E_{22}}} \right.$$

$$+ (n\psi)^4 \sqrt{\frac{E_{22}}{E_{11}}} + 4(mn\psi)^2 \frac{G_{12}(1-\nu_{12}\nu_{21})}{\sqrt{E_{11}E_{22}}}$$

$$\left. -(m\psi)^2\left(1-\frac{c}{2}\right)k_b \right\} A_{mn} \qquad (8)$$

$$-\frac{ck_b}{4\psi}\sum_{q=1}^{\infty}\frac{A_{mq}m^2 nq}{(n^2-q^2)^2}$$

$$+k_s\sum_{p=1}^{\infty}\sum_{q=1}^{\infty}\frac{A_{pq}mnpq}{(p^2-m^2)(n^2-q^2)} = 0$$

Eq. (8) involves three unknowns kb, ks, and the plate aspect ratio a/b. If we neglect the terms involving kb then the equation reduced to the buckling problem of plate under in-plane shear force, and if we neglect the term involving ks then the equation will be the solution of the buckling problem for plate subjecting linearly distributed edge force. Both of these two cases of the problem had been discussed by authors.

Now, Eq. (8) is expanded up to the ten terms, and the constants we used here are A_{11}, A_{12}, A_{13}, A_{21}, A_{22}, A_{23}, A_{31}, A_{32}, A_{33}, and A_{41}. Then ten simultaneous linear equations can be written in a matrix form.

Since the elements of right hand side of equa-

$$\begin{vmatrix}
M_{11} & -\frac{k_b c}{18\psi} & 0 & 0 & -\frac{4k_s}{9} & 0 & 0 & 0 & 0 & 0 \\
-\frac{k_b c}{18\psi} & M_{12} & -\frac{3k_b c}{50\psi} & \frac{4k_s}{9} & 0 & -\frac{4k_s}{5} & 0 & 0 & 0 & \frac{8k_s}{45} \\
0 & -\frac{3k_b c}{50\psi} & M_{13} & 0 & \frac{4k_s}{5} & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{4k_s}{9} & 0 & M_{21} & -\frac{2k_b c}{9\psi} & 0 & 0 & -\frac{4k_s}{5} & 0 & 0 \\
-\frac{4k_s}{9} & 0 & \frac{4k_s}{5} & -\frac{2k_b c}{9\psi} & M_{22} & -\frac{6k_b c}{25\psi} & \frac{4k_s}{5} & 0 & -\frac{36k_s}{25} & 0 \\
0 & -\frac{4k_s}{5} & 0 & 0 & -\frac{6k_b c}{25\psi} & M_{23} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{4k_s}{5} & 0 & M_{31} & -\frac{k_b c}{2\psi} & 0 & 0 \\
0 & 0 & 0 & -\frac{4k_s}{5} & 0 & 0 & -\frac{k_b c}{2\psi} & M_{32} & -\frac{27k_b c}{50\psi} & \frac{8k_s}{7} \\
0 & 0 & 0 & 0 & -\frac{36k_s}{25} & 0 & 0 & -\frac{27k_b c}{50\psi} & M_{33} & 0 \\
0 & \frac{8k_s}{45} & 0 & 0 & 0 & 0 & 0 & \frac{8k_s}{7} & 0 & M_{41}
\end{vmatrix}$$

tions are all zero, the determinant of coefficient matrix of the equations must be vanished to get the solution other than the trivial one. In the determinant of coefficient matrix as shown above, the detail mathematical expression for the main diagonal elements M_{ij}' s are given in Appendix. By expanding the determinant one can obtain the characteristic equation which is a function of ks, k_b, and the plate aspect ratio rm psi `=` a/b. For finding the value of k_s and k_b at each value of rm psinumerical analysis technique such as the secant method is used.

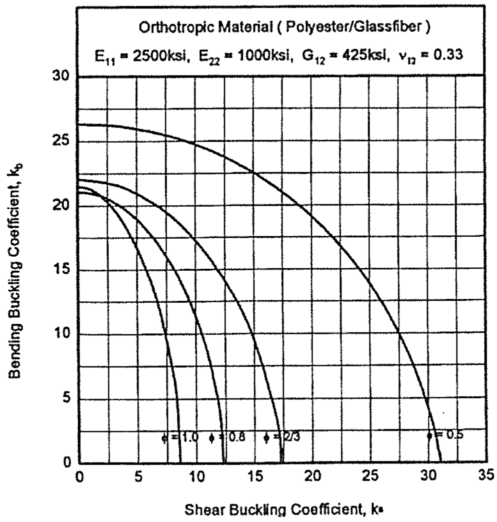

Fig. 3. Variation of kb at each ks (orthotropic material)

To verify the accuracy of results orthotropic material properties (MMFG, 1989)[4] were replaced with isotropic ones. Identical results given in Way(1936)[2], Bleich(1952)[5], Timoshenko(1961)[6], Bulson(1969) [7], and Galambos(1988)[8] are obtained. The graphical forms of results are shown in Fig. 3 for othotropic plates and Fig. 4 for isotropic plates, respectively.

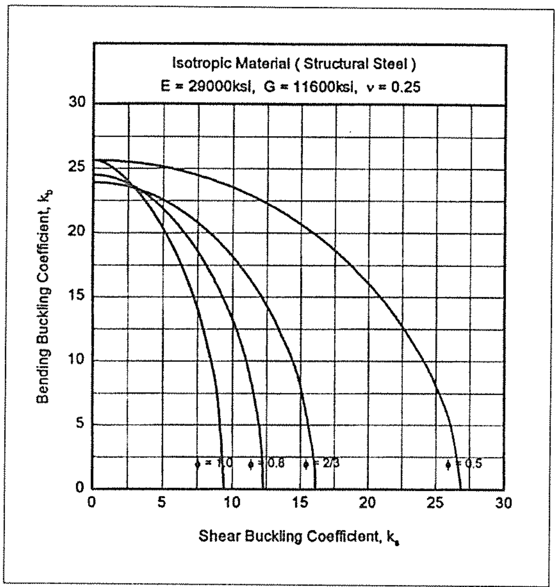

Fig. 4 Variation of kb at each ks(isotropic material)

As shown in Fig. 3 and Fig. 4, the presence of the in-plane shear forces is found to reduce the maximum critical compressive stress as expected.

3. In-plane Shear and Bending Interaction

3.1. In-plane Shear Buckling Strength

As given in Eq. (8) and shown in Fig. 3 for the orthotropic plate and Fig. 4 for the isotropic plate, we discussed the elastic buckling problems for a simply supported rectangular plate under simultaneously acting in-plane pure shear and bending. If we neglect the bending terms in Eq. (8) we may obtain the solution for a plate under in-plane pure shear. In shear buckling, complexity involved due to the change of buckling modes which are symmetric and antisymmetric (Way, 1936[2], Yoon et al. 1998[1]).

But for the design of web plate it is desirable to suggest simple form of equation.

From the investigation equations are proposed by curve fitting technique following Timoshenko and Seydel:For the isotropic plate under in-plane pure shear:

$$k_s = 5.7 + \frac{3.7}{(a/b)^2} \quad \text{for } a/b \le 1 \tag{9a}$$

$$k_s = 3.7 + \frac{5.7}{(a/b)^2} \quad \text{for } a/b \ge 1 \tag{9b}$$

For the orthotropic plate under in-plane pure shear:

$$k_s = 4.1 + \frac{4.5}{(a/b)^2} \quad \text{for } a/b \le 1 \tag{10a}$$

$$k_s = 4.5 + \frac{4.1}{(a/b)^2} \quad \text{for } a/b \ge 1 \tag{10b}$$

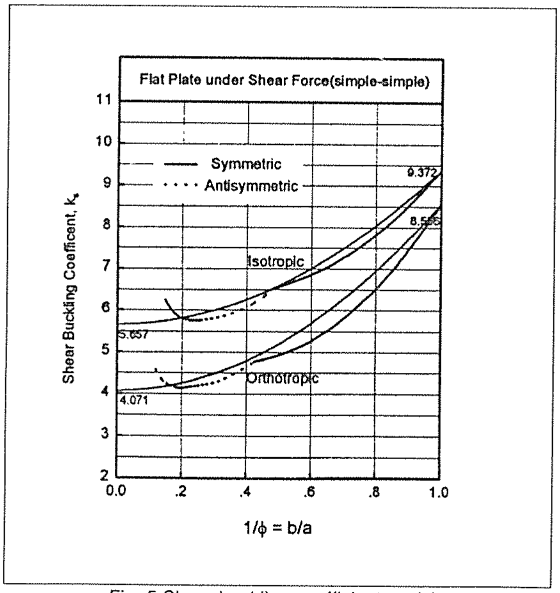

Fig. 5 Shear buckling coefficient vs. b/a

In the formulation of these equations, the energy method suggested by Stowell is used and the values used herein is shown in Fig. 5.

Due to the approximation involved in the energy method, some amount of errors (less than 7 %) is involved. In addition, the elastic shear buckling analysis of infinitely long plate as shown in Fig. 6 is also necessary.

In this plate long edges are assumed to be elastically restrained and the effect (if $\varepsilon = 0$, simply supported; if $\varepsilon = \infty$, fixed support) provided by restraining medium is taken into account. This result for the isotropic and orthotropic plates is shown in Fig. 7, respectively(Jung et al. 1998)[9].

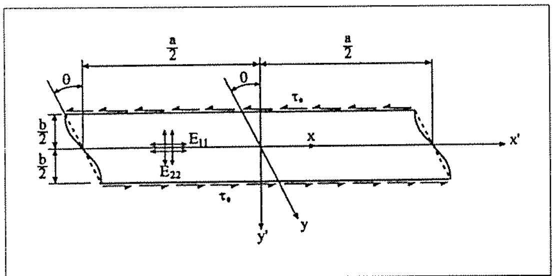

Fig. 6. Infinitely long plate under shear

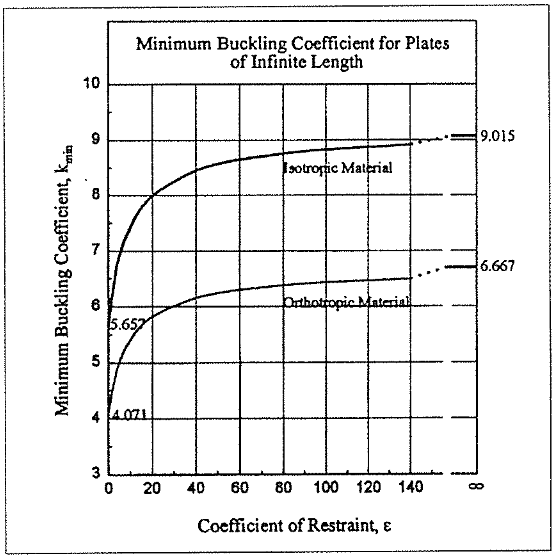

Fig. 7. Minimum buckling coefficient for infinitely long plate

3.2. Simultaneously Acting Compression and Bending Buckling Strength

Detail discussions with results for the analysis of a simply supported rectangular isotropic and orthotropic plates are presented by Yoon et al. (1998)[1]. In order to consider the compression effect, numerical factor c is adopted as shown in Fig. 2. The minimum buckling coefficient kb with respect to the various values of c for the isotropic and orthotropic plates is also given in a paper by Yoon et al. (1998)[1].

3.3. Shear Combined with Bending Buckling Strength

To investigate the interaction between in-plane shear and bending forces, bending and shear interaction curves for orthotropic and isotropic plates are drawn and shown in Fig. 8 and Fig. 9, respectively. As can be seen in Fig. 8 and Fig. 9, the forms of an interaction curves between k_b / $k_{b\text{-}cr}$ and $k_s/k_{s\text{-}cr}$ are shown for various values of plate aspect ratio ψ between 0.5 to 1.0.

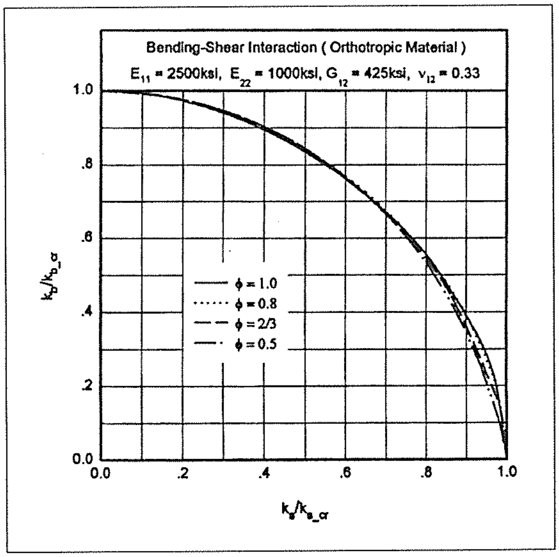

Fig. 8. $k_b/k_{b\text{-}cr}$ vs. $k_s/k_{s\text{-}cr}$(orthotropic material)

The curves, all lie within a narrow zone and for design purposes, are normally replaced by a single curve in the form of a quadrant of a circle (Bleich, 1952[5]; Bulson, 1969[6]). Thus:

$$\left(\frac{\sigma}{\sigma_{cr}}\right)^2 + \left(\frac{\tau}{\tau_{cr}}\right)^2 = 1.0 \tag{11}$$

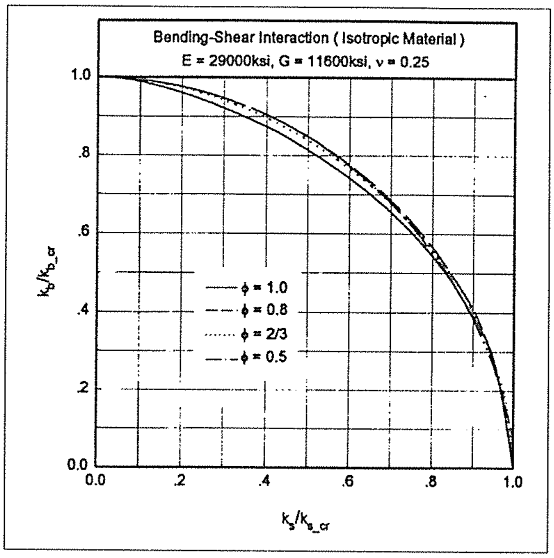

Fig. 9. $k_b/k_{b\text{-}cr}$ vs. $k_s/k_{s\text{-}cr}$(isotropic material)

where rm σ_{cr} and rm τ_{cr} are the buckling stresses due to pure bending and pure in-plane shear, and σ and rm τ are the magnitudes of pure bending and pure in-plane shear stresses acting on the plate. More detail discussions relating to the design formulas for the plates are given by Bleich(1952)[5].

4. Discussion and Conclusion

In this paper the results of analytical elastic buckling analyses for a simply supported rectangular orthotropic plate subjected to in-plane shear force (pure shear) and subjected to bending force (actually bending and compression combined), respectively, are presented. Rayleigh-Ritz method is employed in the solution of the problems. The set of homogeneous linear simultaneous equations obtained were expanded up to the ten terms.

The equations derived in this study were verified by replacing isotropic material properties instead of orthotropic ones and the ensuing equations were compared with existing equations which were derived for a plate with isotropic material. Identical results were observed.

Using derived equations, graphical form of results was suggested so that the elastic buckling strength under in-plane shear and bending could be evaluated. Based on the

results simple design formulas are suggested.

The problems discussed in this paper are solved analytically by the use of Rayleigh-Ritz method. Thus, for the completeness of the investigation, experimental part of the study must be performed. In addition, in the analytical investigation, further research on the material nonlinearity, hygrothermal effect, and residual stress effect should also be conducted.

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Appendix

Main diagonal elements M_{ij}'s are:

$$M_{11} = \frac{\pi^2}{32\phi^3}\left\{\sqrt{\frac{E_{11}}{E_{22}}} + 2\nu_{21}\phi^2\sqrt{\frac{E_{11}}{E_{22}}} + \phi^4\sqrt{\frac{E_{22}}{E_{11}}} + 4\phi^2\frac{G_{12}(1-\nu_{12}\nu_{21})}{\sqrt{E_{11}E_{22}}} - \phi^2 k_b\left(1-\frac{c}{2}\right)\right\}$$

$$M_{12} = \frac{\pi^2}{32\phi^3}\left\{\sqrt{\frac{E_{11}}{E_{22}}} + 8\nu_{21}\phi^2\sqrt{\frac{E_{11}}{E_{22}}} + 16\phi^4\sqrt{\frac{E_{22}}{E_{11}}} + 16\phi^2\frac{G_{12}(1-\nu_{12}\nu_{21})}{\sqrt{E_{11}E_{22}}} - \phi^2 k_b\left(1-\frac{c}{2}\right)\right\}$$

$$M_{13} = \frac{\pi^2}{32\phi^3}\left\{\sqrt{\frac{E_{11}}{E_{22}}} + 18\nu_{21}\phi^2\sqrt{\frac{E_{11}}{E_{22}}} + 81\phi^4\sqrt{\frac{E_{22}}{E_{11}}} + 36\phi^2\frac{G_{12}(1-\nu_{12}\nu_{21})}{\sqrt{E_{11}E_{22}}} - \phi^2 k_b\left(1-\frac{c}{2}\right)\right\}$$

$$M_{21} = \frac{\pi^2}{32\phi^3}\left\{16\sqrt{\frac{E_{11}}{E_{22}}} + 8\nu_{21}\phi^2\sqrt{\frac{E_{11}}{E_{22}}} + \phi^4\sqrt{\frac{E_{22}}{E_{11}}} + 16\phi^2\frac{G_{12}(1-\nu_{12}\nu_{21})}{\sqrt{E_{11}E_{22}}} - 4\phi^2 k_b\left(1-\frac{c}{2}\right)\right\}$$

$$M_{22} = \frac{\pi^2}{32\phi^3}\left\{16\sqrt{\frac{E_{11}}{E_{22}}} + 32\nu_{21}\phi^2\sqrt{\frac{E_{11}}{E_{22}}} + 16\phi^4\sqrt{\frac{E_{22}}{E_{11}}} + 64\phi^2\frac{G_{12}(1-\nu_{12}\nu_{21})}{\sqrt{E_{11}E_{22}}} - 4\phi^2 k_b\left(1-\frac{c}{2}\right)\right\}$$

$$M_{23} = \frac{\pi^2}{32\phi^3}\left\{16\sqrt{\frac{E_{11}}{E_{22}}} + 72\nu_{21}\phi^2\sqrt{\frac{E_{11}}{E_{22}}} + 81\phi^4\sqrt{\frac{E_{22}}{E_{11}}} + 144\phi^2\frac{G_{12}(1-\nu_{12}\nu_{21})}{\sqrt{E_{11}E_{22}}} - 4\phi^2 k_b\left(1-\frac{c}{2}\right)\right\}$$

$$M_{31} = \frac{\pi^2}{32\phi^3}\left\{81\sqrt{\frac{E_{11}}{E_{22}}} + 18\nu_{21}\phi^2\sqrt{\frac{E_{11}}{E_{22}}} + \phi^4\sqrt{\frac{E_{22}}{E_{11}}} + 36\phi^2\frac{G_{12}(1-\nu_{12}\nu_{21})}{\sqrt{E_{11}E_{22}}} - 9\phi^2 k_b\left(1-\frac{c}{2}\right)\right\}$$

$$M_{32} = \frac{\pi^2}{32\phi^3}\left\{81\sqrt{\frac{E_{11}}{E_{22}}} + 72\nu_{21}\phi^2\sqrt{\frac{E_{11}}{E_{22}}} + 16\phi^4\sqrt{\frac{E_{22}}{E_{11}}} + 144\phi^2\frac{G_{12}(1-\nu_{12}\nu_{21})}{\sqrt{E_{11}E_{22}}} - 9\phi^2 k_b\left(1-\frac{c}{2}\right)\right\}$$

$$M_{33} = \frac{\pi^2}{32\phi^3}\left\{81\sqrt{\frac{E_{11}}{E_{22}}} + 162\nu_{21}\phi^2\sqrt{\frac{E_{11}}{E_{22}}} + 81\phi^4\sqrt{\frac{E_{22}}{E_{11}}} + 324\phi^2\frac{G_{12}(1-\nu_{12}\nu_{21})}{\sqrt{E_{11}E_{22}}} - 9\phi^2 k_b\left(1-\frac{c}{2}\right)\right\}$$

$$M_{41} = \frac{\pi^2}{32\phi^3}\left\{256\sqrt{\frac{E_{11}}{E_{22}}} + 32\nu_{21}\phi^2\sqrt{\frac{E_{11}}{E_{22}}} + \phi^4\sqrt{\frac{E_{22}}{E_{11}}} + 64\phi^2\frac{G_{12}(1-\nu_{12}\nu_{21})}{\sqrt{E_{11}E_{22}}} - 16\phi^2 k_b\left(1-\frac{c}{2}\right)\right\}$$

Unit Conversion:

1 ksi = 1000 lb/in^2 = 70.31 kg/cm^2
1 lb = 0.4536 kg
1 in = 2.54 cm