論文

Stability of Simply Supported Orthotropic Rectangular Plates under In-plane Shear and Bending Forces

S.J. Yoon*, B.H. Park** and S.K. Jeong***

면내 전단과 휨을 받는 단순지지된 직교이방성 직사각형 판의 좌굴

윤순종* · 박봉현** · 정상균***

초 록

면내 전단과 휨을 각각 받는 4변이 단순지지된 직교이방성 직사각형 판의 좌굴거동에 대한 해석적 연구결과이다. 펄트루젼 방법으로 생산된 유리섬유보강 플래스틱을 직교이방성으로 가정하였다. 등방성 판의 좌굴해석을 위해 개발된 기존의 이론적 해를 직교이방성 재료의 역학적 성질을 고려하여 좌굴해석 할 수 있도록 확장하였다. 이론식의 해를 구하는데 Rayleigh-Ritz법을 적용하였으며, 면내 전단력과 면내 휨모멘트를 받는 직교이방성 판의 좌굴강도를 구할 수 있도록 해석결과를 그래프 형태로 제시하였다.

ABSTRACT

Presented in this paper are results of an analytical investigation pertaining to the buckling behavior of simply supported orthotropic rectangular plates under separately acting in-plane shear and bending forces. Pultruded fiber reinforced plastics are assumed to be specially orthotropic. The existing analytical solution developed for the isotropic plates is extended so that the orthotropic material properties can be taken into account in the plate buckling analyses. For the solution of the problems Rayleigh-Ritz method is employed. Graphical forms of results for finding the buckling strength of plates under in-plane shear and bending forces are presented.

1. Introduction

In recent years, there has been a greatly increased demand for the use of fiber reinforced plastic (FRP) composite structural shapes in new civil engineering construction and in the rehabilitation and strengthening of existing structures. Such a trend is expected to continue due to increasing requirements for lightweight, high specific stiffness and strength, and nonconducting and noncorroding materials. Many companies in developed countries are currently producing FRP structural shapes such as I-shapes, channels, angles, tees, and tubular shapes (MMFG,

* 홍익대학교 공과대학 토목공학과 조교수
** 홍익대학교 대학원 기계공학과 박사과정
*** 홍익대학교 대학원 토목공학과 박사과정

1989)[1]. Most of the companies producing FRP structural shapes are adopting the pultrusion process which is recognized as the most cost effective manufacturing process. In the pultrusion process, continuous reinforcing fibers with other additional fabric layers are pulled from creels and are passed through a resin bath where fibers are impregnated with polymer resin. The saturated fibers are drawn through a preforming and heating die in which polymerization into a hardened form occurs. The hardened structural shapes are pulled and cut to a desired length.

Although FRP structural shapes are readily available, structural engineers are reluctant to design with this material due to the lack of reliable design criteria (Zureick, 1997)[2]. Thus it is necessary to investigate the mechanical behavior of these structural components under various loading conditions. This paper presents the analytical study results of buckling behavior of web plate of I shaped girder made of the fiber reinforced plastic composite materials.

1-1. Objective

In this paper, we present the results of an analytical investigation pertaining to the short-term compression behavior of separately acting in-plane shear and bending forces. In the design of bridges, ships, automobiles, and aircraft structures, problems arise involving the stability of rectangular plates with various in-plane edge loading. With loading higher than a certain critical value, lateral deflection from the initial plane of the plate takes place. Sometimes, for a structure to carry a load higher than the critical value may be permissible, but a knowledge of the critical load is always desirable.

The FRP girder is assumed to be composed of specially orthotropic homogeneous materials that could be characterized with four independent elastic constants: the longitudinal Young's modulus E_{11}, the transverse Young's modulus E_{22}, the in-plane shear modulus G_{12}, and the major Poisson's ratio ν_{12}. The existing analytical solution developed for the isotropic plates must be modified so that the orthotropic material properties can be taken into account in the buckling analyses of the web of plate girder.

1-2. Previous Work

In relation to the problems discussed we briefly review the literatures focused on the isotropic and orthotropic plates under in-plane shear and bending forces. Most of these literatures are reviewed by Bleich(1952)[3], Timoshenko(1961)[4], Bulson(1969)[5], and Galambos(1988)[6].

For the isotropic plates, Boobnoff(1914) investigated the simply supported rectangular plate under combined bending and compressive stresses acting in the plane of the plate on two opposite edges. Timoshenko(1921, 1935) was the first to present a practical solution of the stability problem of rectangular plates acting in shear by applying the energy method. He applied the energy method also to the determination of the critical stress of simply supported rectangular plates under bending and compressive stresses and extended the investigation to the case of combined in-plane shear and pure bending stresses. The accuracy of the Timoshenko's results for simply supported plates was improved by Bergmann and Reissner(1932) and by Seydel(1933)[7]. Stein and Neff(1947)[7] determined the critical in-plane shear stresses more accurately than previous authors by considering symmetric and antisymmetric buckling modes.

Buckling under nonuniformly distributed compressive stresses acting on two opposite sides of the plate was considered by Nölke(1936, 1937)[8], who treated plate with fixed edges. Plates under combined bending and in-plane shear stresses were studied by Stein(1934), who gave tables showing the interaction between the critical longitudinal stresses and the critical in-plane shear stresses. Papers by Batdorf and Stein(1947) and by Stowell and Schwartz are also devoted to the problem of plates under combined in-plane shear and uniform longitudinal stresses[7]. Southwell and Skan(1924) have treated the special case of an infinitely long plate strip with edge shearing force[7].

For the orthotropic plates, Lekhnitskii(1947, 1984)[9] reviewed the works relating to the buckling of plates under in-plane shear, bending, and combined in-plane shear and bending forces. The orthotropic plate buckling analysis was performed by the energy method developed by Timoshenko. Most of the results are sound for the practical applications but those results are approximated for simplicity.

2. Theoretical Derivations

In this paper we solved the simply supported orthotropic rectangular plates under in-plane shear and bending forces by employing the Rayleigh-Ritz method. The method adopted in this study was originally applied by Way(1936)[10] to the isotropic plates. By the application of this method desired accuracy can be obtained simply by the series expansion with more terms.

According to the classical orthotropic plate theory (Lekhnitskii, 1984)[9], the strain energy of bending of orthotropic rectangular thin plates whose boundaries are simply supported is given by:

$$U=\frac{1}{2}\int_0^b\int_0^a\left\{D_{11}\left(\frac{\partial^2 w}{\partial x^2}\right)^2+2D_{11}\nu_{21}\left(\frac{\partial^2 w}{\partial x^2}\right)\left(\frac{\partial^2 w}{\partial y^2}\right)+D_{22}\left(\frac{\partial^2 w}{\partial y^2}\right)^2+4D_{66}\left(\frac{\partial^2 w}{\partial x\partial y}\right)^2\right\}dxdy \quad (1)$$

The work done by the bending and in-plane shear forces is given by, respectively:

$$V_b=\frac{1}{2}\sigma_0 t\int_0^b\int_0^a\left(1-\frac{c}{b}y\right)\left(\frac{\partial w}{\partial x}\right)^2 dxdy \quad (2)$$

$$V_s=-\tau_0 t\int_0^b\int_0^a\left(\frac{\partial w}{\partial x}\right)\left(\frac{\partial w}{\partial y}\right)dxdy \cdots\cdots\cdots (3)$$

In Eqs. (1) to (3) w is the out-of-plane deflection due to in-plane shear and bending forces, respectively. In Eq. (1) D_{11} and D_{22} are the flexural rigidities in 1-1 and 2-2 material property directions and D_{66} is the twisting rigidity of plate in 1-2 direction, and ν_{12} and ν_{21} are the major and minor Poisson' s ratios, and they are defined by, respectively:

$$D_{11}=\frac{E_{11}t^3}{12(1-\nu_{12}\nu_{21})},\quad D_{22}=\frac{E_{22}t^3}{12(1-\nu_{12}\nu_{21})},$$

$$D_{66}=\frac{G_{12}t^3}{12},\quad \nu_{21}=\nu_{12}\frac{E_{22}}{E_{11}}, \qquad (4a,b,c,d)$$

These material property directions, coordinate axes, and the bending and in-plane shear stresses acting on the plate are shown in Fig. 1 and Fig. 2, respectively. Plate width and length are b and a, and the plate thickness t is assumed to be uniform throughout the plate.

As shown in Eqs. (1) to (3) the orthotropic

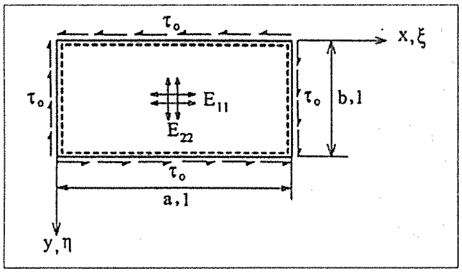

Fig. 1. Plate under In-plane Shear Stresses

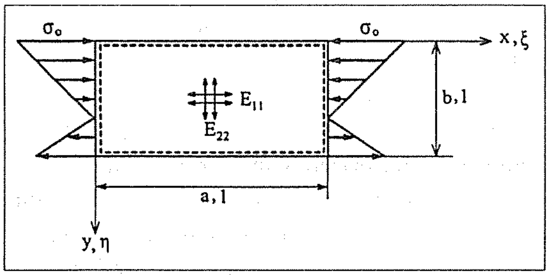

Fig. 2. Plate under Bending Stresses

material properties are only involved in the expression for the strain energy of plate bending.

For the efficiency of numerical calculation of the results, the nondimensionalized parameters are adopted as:

$$\xi=\frac{x}{a},\ \eta=\frac{y}{b},\ \psi=\frac{a}{b} \tag{5,a,b,c}$$

Upon substitution of Eqs. (5,a,b,c) into Eqs. (1) to (3) and using the chain-rule, we may transform the equations into the following form, respectively:

$$U=\frac{ab}{2}\int_0^1\int_0^1\left\{\frac{D_{11}}{a^4}\left(\frac{\partial^2 w}{\partial\xi^2}\right)^2+\frac{2D_{11}\nu_{21}}{a^2b^2}\left(\frac{\partial^2 w}{\partial\xi^2}\right)\left(\frac{\partial^2 w}{\partial\eta^2}\right)+\frac{D_{22}}{b^4}\left(\frac{\partial^2 w}{\partial\eta^2}\right)^2+\frac{4D_{66}}{a^2b^2}\left(\frac{\partial^2 w}{\partial\xi\partial\eta}\right)^2\right\}d\xi d\eta \tag{6}$$

$$V_b=\frac{ab}{2}\sigma_0 t\int_0^1\int_0^1\frac{1}{a^2}(1-c\eta)\left(\frac{\partial w}{\partial\xi}\right)^2 d\xi d\eta \tag{7}$$

$$V_s=-\tau_0 t\int_0^1\int_0^1\left(\frac{\partial w}{\partial\xi}\right)\left(\frac{\partial w}{\partial\eta}\right)d\xi d\eta \tag{8}$$

The deflection w of the simply supported plate can be taken in the form of double trigonometric sine series following Navier.

$$w=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}A_{mn}\sin m\pi\xi\sin n\pi\eta \tag{9}$$

Differentiating Eq. (9) and substituting into Eqs. (6), (7), and (8) we may obtain the expression for the strain energy U, the work done by in-plane shear stresses V_s, and the work done by bending stresses V_b.

2-1. Elastic Buckling of Plate under In-plane Shear Stresses

Upon substituting Eq. (9) into Eq. (6), as mentioned above, the strain energy U of plate bending is obtained:

$$U=\frac{ab}{8}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}A_{mn}^2\left\{D_{11}\left(\frac{m\pi}{a}\right)^4+2\nu_{21}D_{11}\left(\frac{m\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)^2+D_{22}\left(\frac{n\pi}{b}\right)^4+4D_{66}\left(\frac{m\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)^2\right\} \tag{10}$$

By substitution of Eq. (9) into Eq. (8) we may find the expression for the work done by in-plane shear stresses as following:

$$V_s=4k_s\left(\frac{\pi}{b}\right)^2\sqrt{D_{11}D_{22}}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{p=1}^{\infty}\sum_{q=1}^{\infty}A_{mn}A_{pq}\frac{mnpq}{(p^2-m^2)(n^2-q^2)} \tag{11}$$

where ks is the in-plane shear buckling coefficient of plate and it is defined as Eq. (12), and in Eq. (11) m+p and n+q are odd numbers.

$$k_s=\frac{\tau_0 t b^2}{\pi^2\sqrt{D_{11}D_{22}}} \tag{12}$$

Therefore, the in-plane shear buckling stress τ_{cr} can be written in the form following Timoshenko(1961)[4]:

$$\tau_{cr}=\frac{k_s\pi^2\sqrt{D_{11}D_{22}}}{b_2 t}=\frac{k_s\pi^2\sqrt{E_{11}E_{22}}}{12(1-\nu_{12}\nu_{21})\left(\frac{b}{t}\right)^2} \tag{13}$$

The total potential energy Π is given by:

$$\Pi=U-V_s \tag{14}$$

The coefficient A_{mn} must be chosen to make the value of k_s a minimum. Using the Rayleigh-Ritz method, the minimization of k_s with respect to each A_{mn} results in the set of homogeneous linear equations represented by the following equation:

$$\frac{\pi_2 A_{mn}}{32k_s\psi^3}\left\{m^4\sqrt{\frac{E_{11}}{E_{22}}}+2\nu_{21}m^2n^2\psi^2\sqrt{\frac{E_{11}}{E_{22}}}+n^4\psi^4\sqrt{\frac{E_{22}}{E_{11}}}\right.$$

$$
\left. +4m^2n^2\psi^2\frac{G_{12}(1-\nu_{12}\nu_{21})}{\sqrt{E_{11}E_{22}}}\right\}
$$

$$
-\sum_{p=1}^{\infty}\sum_{q=1}^{\infty}\frac{A_{pq}mnpq}{(p^2-m^2)(n^2-q^2)}=0 \qquad (15)
$$

where $\psi=a/b$, $m=1, 2, 3, \cdots$, $n=1, 2, 3, \cdots$, and $m+p$ and $n+q$ are odd numbers.

Each of the equations represented by Eq. (15) is associated with a specific pair of values of m and n. Since m+p and n+q are both odd, m+p+n+q must be even. If m+n is even, then p+q must also be even; if m+n is odd, p+q must also be odd. Each of the homogeneous linear Eqs. (15) can therefore involve only coefficient A_{ij} for which i+j is either even or odd. Therefore the set of Eqs. (15) can be divided into two independent equations which can be solved separately, one group consisting of equations in which i+j is odd and the other group consisting equations i+j is even. The set of equations in which i+j is even corresponds to symmetric buckling mode, and the set in which i+j is odd corresponds to antisymmetric buckling mode. Ten equations in ten unknowns were solved for k_s for each type of buckling (symmetric and antisymmetric). Ten equations may be written in matrix form. Since the elements of right hand side of equations are all zero, the determinant of the equations must be vanished to get the solution

$$
\begin{vmatrix}
S_{11} & 0 & \frac{4}{9} & 0 & 0 & \frac{8}{45} & 0 & \frac{8}{45} & 0 & 0 \\
0 & S_{13} & -\frac{4}{5} & 0 & 0 & \frac{8}{7} & 0 & -\frac{8}{25} & 0 & 0 \\
\frac{4}{9} & -\frac{4}{5} & S_{22} & -\frac{4}{5} & -\frac{20}{63} & 0 & \frac{36}{25} & 0 & -\frac{20}{63} & \frac{4}{7} \\
0 & 0 & -\frac{4}{5} & S_{31} & 0 & -\frac{8}{25} & 0 & \frac{8}{7} & 0 & 0 \\
0 & 0 & -\frac{20}{63} & 0 & S_{15} & -\frac{40}{27} & 0 & -\frac{8}{63} & 0 & 0 \\
\frac{8}{45} & \frac{8}{7} & 0 & -\frac{8}{25} & -\frac{40}{27} & S_{24} & -\frac{72}{35} & 0 & -\frac{8}{63} & \frac{8}{3} \\
0 & 0 & \frac{36}{25} & 0 & 0 & -\frac{72}{35} & S_{33} & -\frac{72}{35} & 0 & 0 \\
\frac{8}{45} & -\frac{8}{25} & 0 & \frac{8}{7} & -\frac{8}{63} & 0 & -\frac{72}{35} & S_{42} & -\frac{40}{27} & -\frac{40}{49} \\
0 & 0 & -\frac{20}{63} & 0 & 0 & -\frac{8}{63} & 0 & -\frac{40}{27} & S_{51} & 0 \\
0 & 0 & \frac{4}{7} & 0 & 0 & \frac{8}{3} & 0 & -\frac{40}{49} & 0 & S_{35}
\end{vmatrix}
$$

other than the trivial one. By expanding the determinant one can obtain the characteristic equation which is a function of k_s and plate aspect ratio a/b (ψ=a/b).

For finding the value of k_s at each value of ψ numerical analysis technique such as the secant method is used. A representative determinant for a group of equations in which i+j is even (symmetric buckling mode) is shown below.

In the above determinant, detail mathematical expression for the main diagonal elements S_{ij}' s are given in Appendix.

At a plate aspect ratio of 1 the lowest value of ks that satisfies this determinant is less than the lowest value of k_s obtained from any tenth-order determinant in which i+j is odd. A representative determinant for a group of equations in which i+j is odd (antisymmetric buckling mode) is also given.

$$
\begin{vmatrix}
T_{12} & -\frac{4}{9} & \frac{4}{5} & 0 & -\frac{8}{45} & \frac{8}{25} & 0 & 0 & -\frac{4}{35} & \frac{36}{175} \\
-\frac{4}{9} & T_{21} & 0 & \frac{4}{5} & 0 & 0 & \frac{20}{63} & \frac{8}{63} & 0 & 0 \\
\frac{4}{5} & 0 & T_{23} & -\frac{36}{25} & 0 & 0 & -\frac{4}{7} & \frac{40}{49} & 0 & 0 \\
0 & \frac{4}{5} & -\frac{36}{25} & T_{32} & -\frac{8}{7} & \frac{72}{35} & 0 & 0 & -\frac{4}{9} & \frac{4}{5} \\
-\frac{8}{45} & 0 & 0 & -\frac{8}{7} & T_{41} & 0 & \frac{40}{27} & \frac{16}{27} & 0 & 0 \\
\frac{8}{25} & 0 & 0 & \frac{72}{35} & 0 & T_{43} & -\frac{8}{3} & \frac{80}{21} & 0 & 0 \\
0 & \frac{20}{63} & -\frac{4}{7} & 0 & \frac{40}{27} & -\frac{8}{3} & T_{52} & 0 & -\frac{20}{11} & \frac{36}{11} \\
0 & \frac{8}{63} & \frac{40}{49} & 0 & \frac{16}{27} & \frac{80}{21} & 0 & T_{54} & -\frac{8}{11} & -\frac{360}{77} \\
-\frac{4}{35} & 0 & 0 & -\frac{4}{9} & 0 & 0 & -\frac{20}{11} & -\frac{8}{11} & T_{61} & 0 \\
\frac{36}{175} & 0 & 0 & \frac{4}{5} & 0 & 0 & \frac{36}{11} & -\frac{360}{77} & 0 & T_{63}
\end{vmatrix}
$$

The buckling coefficient ks with respect to each value of a/b is shown in Fig. 3.

2-2. Elastic Buckling of Plate under Bending Stresses

Upon substitution of Eq. (9) into Eq. (7) we can find the work done by bending forces and the ensuing equation is given in Eq. (16).

$$
V_b=\frac{ab\sigma_0 t}{8}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}A_{mn}^2\left(\frac{m\pi}{a}\right)^2-\frac{\sigma_0 tac}{4b}\sum_{m=1}^{\infty}\left(\frac{m\pi}{a}\right)^2
$$

$$\left\{\frac{b^2}{4}\sum_{n=1}^{\infty}A_{mn}^2-\frac{4b^2}{\pi^2}\sum_{n=1}^{\infty}\sum_{q=1}^{\infty}A_{mn}A_{mq}\frac{nq}{(n^2-q^2)^2}\right\} \tag{16}$$

Since the strain energy of plate bending is given in Eq. (10), the total potential energy of this case may be written and using the same procedure discussed earlier we may find the set of homogeneous linear equations represented by the following equation:

$$\left\{m^4\sqrt{\frac{E_{11}}{E_{22}}}+2\nu_{21}m^2n^2\psi^2\sqrt{\frac{E_{11}}{E_{22}}}+n^4\psi^4\sqrt{\frac{E_{22}}{E_{11}}}\right.$$
$$\left.+4m^2n^2\psi^2\frac{G_{12}(1-\nu_{12}\nu_{21})}{\sqrt{E_{11}E_{22}}}\right\}A_{mn}$$
$$-\frac{8cm^2\psi^2}{\pi^2}k_b\sum_{q=1}^{\infty}\frac{A_{mq}nq}{(n^2-q^2)^2}=0 \tag{17}$$

where buckling coefficient k_b is defined as:

$$k_b=\frac{\sigma_0 tb^2}{\pi^2\sqrt{D_{11}D_{22}}} \tag{18}$$

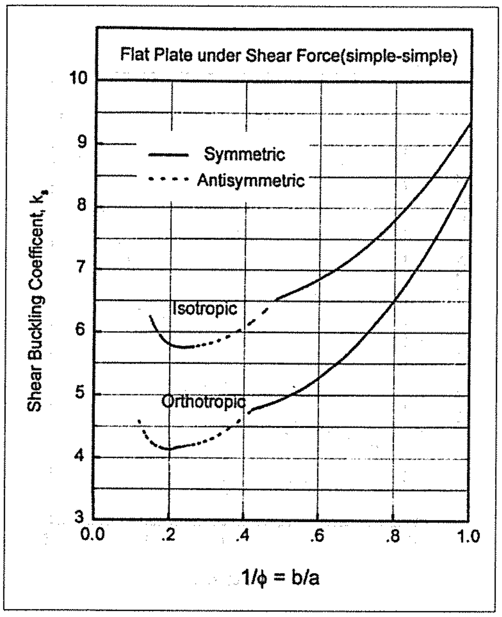

Fig. 3. In-plane Shear Buckling Coefficients, k_s

Therefore the buckling stress σ_{cr} due to bending forces can be calculated from the following equation:

$$\sigma_{cr}=\frac{k_b\pi^2\sqrt{D_{11}D_{22}}}{b^2t}=\frac{k_b\pi^2\sqrt{E_{11}E_{22}}}{12(1-\nu_{12}\nu_{21})\left(\frac{b}{t}\right)^2} \tag{19}$$

Now, we have to expand the Eq. (17) in which any integer for m and n can be chosen but $n\pm q$ must be odd number. In this study the Eq. (17) was expanded up to the ten terms. Then, the set of ten homogeneous linear simultaneous equations may be written in matrix form. Since the equation is homogeneous the nontrivial solution exists only when the determinant of the coefficient matrix vanished. A representative determinant for the equations in which $n\pm q$ is odd is given.

R_{11}	$-\frac{k_\beta}{9}$	0	0	0	0	0	0	0	0
$-\frac{k_\beta}{9}$	R_{12}	$-\frac{3k_\beta}{25}$	0	0	0	0	0	0	0
0	$-\frac{3k_\beta}{25}$	R_{13}	0	0	0	0	0	0	0
0	0	0	R_{21}	$-\frac{4k_\beta}{9}$	0	0	0	0	0
0	0	0	$-\frac{4k_\beta}{9}$	R_{22}	$-\frac{12k_\beta}{25}$	0	0	0	0
0	0	0	0	$-\frac{12k_\beta}{25}$	R_{23}	0	0	0	0
0	0	0	0	0	0	R_{31}	$-k_\beta$	0	0
0	0	0	0	0	0	$-k_\beta$	R_{32}	$-\frac{27k_\beta}{25}$	0
0	0	0	0	0	0	0	$-\frac{27k_\beta}{25}$	R_{33}	0
0	0	0	0	0	0	0	0	0	R_{41}

In the determinant, k_β and R_{ij}' s are also given in Appendix. The determinant is a function of plate buckling coefficient k_b and plate aspect ratio a/b. Using the same numerical technique the buckling coefficients k_b at each plate aspect ratio a/b is calculated and the results are represented graphically. As shown in the determinant, the buckling coefficient is also a function of numerical factor c which represents the shape of bending stresses. The numerical factor c is defined in Fig. 4. Uniform compression is applied if c=0 and pure bending forces are applied if c=2. Thus the values of c are varied from 0 to 2. In order to find the buckling coefficient, numerical factor c must be defined prior to the calculation for k_b at each plate aspect ratio a/b.

To verify the accuracy of results orthotropic material properties were replaced with isotropic ones. Identical results given in Bleich(1952)[3], Timoshenko(1961)[4], Bulson(1969)[5], and Galambos(1988)[6] are obtained. The graphical form of results is shown in Fig. 5 for othotropic plates and Fig. 6 for isotropic plates, respectively. The graph for minimum plate buckling coefficient k_b at each numerical factor c is also drawn and shown in Fig. 7 so that the minimum buckling coefficient can be obtained easily according to the various values of numerical factor c.

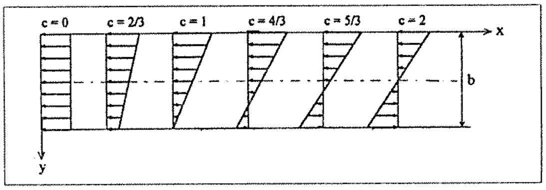

Fig. 4. Numerical Factor c

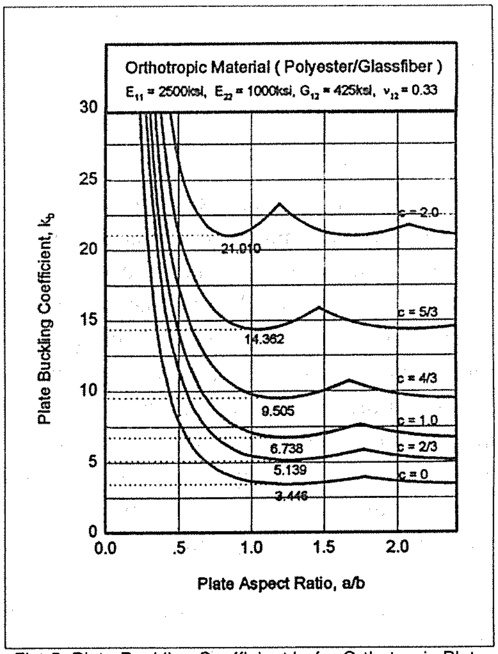

Fig. 5. Plate Buckling Coefficient k_b for Orthotropic Plate

3. Discussion of Results and Conclusion

In this paper the results of analytical elastic buckling analysis for a simply supported rectan-

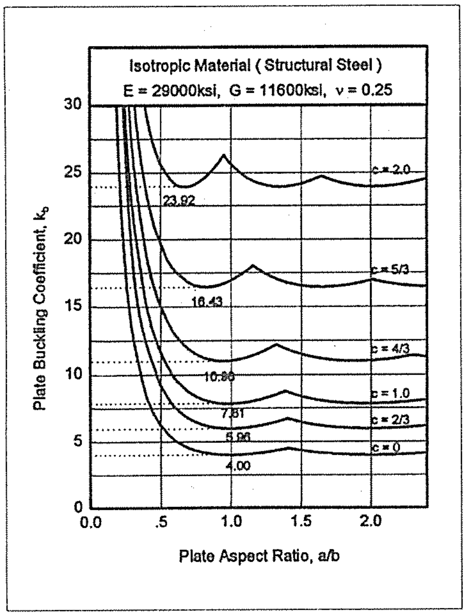

Fig. 6. Plate Buckling Coefficient k_b for Isotropic Plate

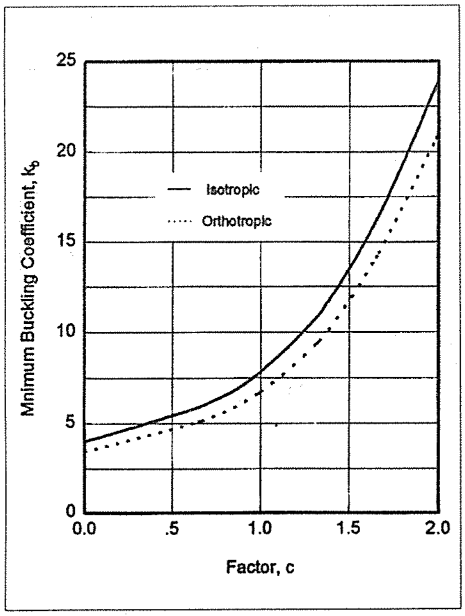

Fig. 7. Minimum Buckling Coefficient k_b vs. Numerical Factor c

gular orthotropic plate subjected to in-plane shear force and subjected to bending force, respectively, are presented. Rayleigh-Ritz method is employed in the solution of the problems. The set of homogeneous linear simultaneous equations obtained were expanded up to the ten terms.

The equations derived in this study were verified by replacing isotropic material properties instead of orthotropic ones and the ensuing equations were compared with existing equations which were derived for a plate with isotropic material. Identical results given in Bulson(1952), Galambos(1988) etc. were observed.

Using derived equations, graphical form of results was suggested so that the elastic buckling strength under in-plane shear and bending could be evaluated.

In this paper we discussed the short-term elastic buckling behavior, so the long-term effects such as creep and shrinkage, temperature and moisture variations(hygrothermal effect), and degradation by the ultra violet ray are not taken into account. In addition, the post-buckling and inelastic buckling strengths are not considered because of the brittle nature of fiber and matrix materials. The above mentioned effects need to be investigated further.

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Appendix

Main diagonal elements S_{ij}' s are:

$$S_{11}=\frac{\pi^2}{32\psi^3 k_s}\left\{\sqrt{\frac{E_{11}}{E_{22}}}+2\nu_{21}\psi^2\sqrt{\frac{E_{11}}{E_{22}}}+\psi^4\sqrt{\frac{E_{22}}{E_{11}}}+4\psi^2\frac{G_{12}(1-\nu_{12}\nu_{21})}{\sqrt{E_{11}E_{22}}}\right\}$$

$$S_{13}=\frac{\pi^2}{32\psi^3 k_s}\left\{\sqrt{\frac{E_{11}}{E_{22}}}+18\nu_{21}\psi^2\sqrt{\frac{E_{11}}{E_{22}}}+81\psi^4\sqrt{\frac{E_{22}}{E_{11}}}+36\psi^2\frac{G_{12}(1-\nu_{12}\nu_{21})}{\sqrt{E_{11}E_{22}}}\right\}$$

$$S_{22}=\frac{16\pi^2}{32\psi^3 k_s}\left\{\sqrt{\frac{E_{11}}{E_{22}}}+2\nu_{21}\psi^2\sqrt{\frac{E_{11}}{E_{22}}}+\psi^4\sqrt{\frac{E_{22}}{E_{11}}}+36\psi^2\frac{G_{12}(1-\nu_{12}\nu_{21})}{\sqrt{E_{11}E_{22}}}\right\}$$

$$S_{31}=\frac{\pi^2}{32\psi^3 k_s}\left\{\sqrt{\frac{E_{11}}{E_{22}}}+18\nu_{21}\psi^2\sqrt{\frac{E_{11}}{E_{22}}}+\psi^4\sqrt{\frac{E_{22}}{E_{11}}}+36\psi^2\frac{G_{12}(1-\nu_{12}\nu_{21})}{\sqrt{E_{11}E_{22}}}\right\}$$

$$S_{15}=\frac{\pi^2}{32\psi^3 k_s}\left\{\sqrt{\frac{E_{11}}{E_{22}}}+50\nu_{21}\psi^2\sqrt{\frac{E_{11}}{E_{22}}}+625\psi^4\sqrt{\frac{E_{22}}{E_{11}}}+100\psi^2\frac{G_{12}(1-\nu_{12}\nu_{21})}{\sqrt{E_{11}E_{22}}}\right\}$$

$$S_{33}=\frac{\pi^2}{32\psi^3 k_s}\left\{\sqrt{\frac{E_{11}}{E_{22}}}+2\nu_{21}\psi^2\sqrt{\frac{E_{11}}{E_{22}}}+\psi^4\sqrt{\frac{E_{22}}{E_{11}}}+4\psi^2\frac{G_{12}(1-\nu_{12}\nu_{21})}{\sqrt{E_{11}E_{22}}}\right\}$$

$$S_{24}=\frac{16\pi^2}{32\psi^3 k_s}\left\{\sqrt{\frac{E_{11}}{E_{22}}}+8\nu_{21}\psi^2\sqrt{\frac{E_{11}}{E_{22}}}+16\psi^4\sqrt{\frac{E_{22}}{E_{11}}}+16\psi^2\frac{G_{12}(1-\nu_{12}\nu_{21})}{\sqrt{E_{11}E_{22}}}\right\}$$

$$S_{42}=\frac{16\pi^2}{32\psi^3 k_s}\left\{16\sqrt{\frac{E_{11}}{E_{22}}}+8\nu_{21}\psi^2\sqrt{\frac{E_{11}}{E_{22}}}+\psi^4\sqrt{\frac{E_{22}}{E_{11}}}+16\psi^2\frac{G_{12}(1-\nu_{12}\nu_{21})}{\sqrt{E_{11}E_{22}}}\right\}$$

$$S_{51}=\frac{\pi^2}{32\psi^3 k_s}\left\{625\sqrt{\frac{E_{11}}{E_{22}}}+50\nu_{21}\psi^2\sqrt{\frac{E_{11}}{E_{22}}}+\psi^4\sqrt{\frac{E_{22}}{E_{11}}}+100\psi^2\frac{G_{12}(1-\nu_{12}\nu_{21})}{\sqrt{E_{11}E_{22}}}\right\}$$

$$S_{35}=\frac{\pi^2}{32\psi^3 k_s}\left\{81\sqrt{\frac{E_{11}}{E_{22}}}+450\nu_{21}\psi^2\sqrt{\frac{E_{11}}{E_{22}}}+625\psi^4\sqrt{\frac{E_{22}}{E_{11}}}+900\psi^2\frac{G_{12}(1-\nu_{12}\nu_{21})}{\sqrt{E_{11}E_{22}}}\right\}$$

Main diagonal elements T_{ij}' s are:

$$T_{12}=\frac{\pi^2}{32\psi^3 k_s}\left\{\sqrt{\frac{E_{11}}{E_{22}}}+8\nu_{21}\psi^2\sqrt{\frac{E_{11}}{E_{22}}}+4\psi^4\sqrt{\frac{E_{22}}{E_{11}}}+16\psi^2\frac{G_{12}(1-\nu_{12}\nu_{21})}{\sqrt{E_{11}E_{22}}}\right\}$$

$$T_{21}=\frac{\pi^2}{32\psi^3 k_s}\left\{16\sqrt{\frac{E_{11}}{E_{22}}}+8\nu_{21}\psi^2\sqrt{\frac{E_{11}}{E_{22}}}+\psi^4\sqrt{\frac{E_{22}}{E_{11}}}+16\psi^2\frac{G_{12}(1-\nu_{12}\nu_{21})}{\sqrt{E_{11}E_{22}}}\right\}$$

$$T_{23}=\frac{\pi^2}{32\psi^3 k_s}\left\{16\sqrt{\frac{E_{11}}{E_{22}}}+72\nu_{21}\psi^2\sqrt{\frac{E_{11}}{E_{22}}}+81\psi^4\sqrt{\frac{E_{22}}{E_{11}}}+144\psi^2\frac{G_{12}(1-\nu_{12}\nu_{21})}{\sqrt{E_{11}E_{22}}}\right\}$$

$$T_{32}=\frac{\pi^2}{32\psi^3 k_s}\left\{81\sqrt{\frac{E_{11}}{E_{22}}}+72\nu_{21}\psi^2\sqrt{\frac{E_{11}}{E_{22}}}+81\psi^4\sqrt{\frac{E_{22}}{E_{11}}}+144\psi^2\frac{G_{12}(1-\nu_{12}\nu_{21})}{\sqrt{E_{11}E_{22}}}\right\}$$

$$T_{41}=\frac{\pi^2}{32\psi^3 k_s}\left\{256\sqrt{\frac{E_{11}}{E_{22}}}+32\nu_{21}\psi^2\sqrt{\frac{E_{11}}{E_{22}}}+\psi^4\sqrt{\frac{E_{22}}{E_{11}}}+64\psi^2\frac{G_{12}(1-\nu_{12}\nu_{21})}{\sqrt{E_{11}E_{22}}}\right\}$$

$$T_{43}=\frac{\pi^2}{32\psi^3 k_s}\left\{256\sqrt{\frac{E_{11}}{E_{22}}}+288\nu_{21}\psi^2\sqrt{\frac{E_{11}}{E_{22}}}+81\psi^4\sqrt{\frac{E_{22}}{E_{11}}}+576\psi^2\frac{G_{12}(1-\nu_{12}\nu_{21})}{\sqrt{E_{11}E_{22}}}\right\}$$

$$T_{52}=\frac{\pi^2}{32\psi^3 k_s}\left\{625\sqrt{\frac{E_{11}}{E_{22}}}+200\nu_{21}\psi^2\sqrt{\frac{E_{11}}{E_{22}}}+16\psi^4\sqrt{\frac{E_{22}}{E_{11}}}+400\psi^2\frac{G_{12}(1-\nu_{12}\nu_{21})}{\sqrt{E_{11}E_{22}}}\right\}$$

$$T_{54}=\frac{\pi^2}{32\psi^3 k_s}\left\{625\sqrt{\frac{E_{11}}{E_{22}}}+800\nu_{21}\psi^2\sqrt{\frac{E_{11}}{E_{22}}}+256\psi^4\sqrt{\frac{E_{22}}{E_{11}}}+1600\psi^2\frac{G_{12}(1-\nu_{12}\nu_{21})}{\sqrt{E_{11}E_{22}}}\right\}$$

$$T_{61}=\frac{\pi^2}{32\psi^3 k_s}\left\{1296\sqrt{\frac{E_{11}}{E_{22}}}+72\nu_{21}\psi^2\sqrt{\frac{E_{11}}{E_{22}}}+\psi^4\sqrt{\frac{E_{22}}{E_{11}}}+144\psi^2\frac{G_{12}(1-\nu_{12}\nu_{21})}{\sqrt{E_{11}E_{22}}}\right\}$$

$$T_{63}=\frac{\pi^2}{32\psi^3 k_s}\left\{1296\sqrt{\frac{E_{11}}{E_{22}}}+648\nu_{21}\psi^2\sqrt{\frac{E_{11}}{E_{22}}}+81\psi^4\sqrt{\frac{E_{22}}{E_{11}}}+1296\psi^2\frac{G_{12}(1-\nu_{12}\nu_{21})}{\sqrt{E_{11}E_{22}}}\right\}$$

Main diagonal elements R_{ij}' s and

$$R_{11}=\left\{\sqrt{\frac{E_{11}}{E_{22}}}+2\nu_{21}\psi^2\sqrt{\frac{E_{11}}{E_{22}}}+\psi^4\sqrt{\frac{E_{22}}{E_{11}}}+4\psi^2\frac{G_{12}(1-\nu_{12}\nu_{21})}{\sqrt{E_{11}E_{22}}}-\psi^2(1-\frac{c}{2})k_b\right\}$$

$$R_{12}=\left\{\sqrt{\frac{E_{11}}{E_{22}}}+8\nu_{21}\psi^2\sqrt{\frac{E_{11}}{E_{22}}}+16\psi^4\sqrt{\frac{E_{22}}{E_{11}}}+16\psi^2\frac{G_{12}(1-\nu_{12}\nu_{21})}{\sqrt{E_{11}E_{22}}}-\psi^2(1-\frac{c}{2})k_b\right\}$$

$$R_{13}=\left\{\sqrt{\frac{E_{11}}{E_{22}}}+18\nu_{21}\psi^2\sqrt{\frac{E_{11}}{E_{22}}}+81\psi^4\sqrt{\frac{E_{22}}{E_{11}}}+36\psi^2\frac{G_{12}(1-\nu_{12}\nu_{21})}{\sqrt{E_{11}E_{22}}}-\psi^2(1-\frac{c}{2})k_b\right\}$$

$$R_{21}=\left\{16\sqrt{\frac{E_{11}}{E_{22}}}+8\nu_{21}\psi^2\sqrt{\frac{E_{11}}{E_{22}}}+\psi^4\sqrt{\frac{E_{22}}{E_{11}}}+16\psi^2\frac{G_{12}(1-\nu_{12}\nu_{21})}{\sqrt{E_{11}E_{22}}}-\psi^2(1-\frac{c}{2})k_b\right\}$$

$$R_{22}=\left\{16\sqrt{\frac{E_{11}}{E_{22}}}+32\nu_{21}\psi^2\sqrt{\frac{E_{11}}{E_{22}}}+16\psi^4\sqrt{\frac{E_{22}}{E_{11}}}+64\psi^2\frac{G_{12}(1-\nu_{12}\nu_{21})}{\sqrt{E_{11}E_{22}}}-4\psi^2(1-\frac{c}{2})k_b\right\}$$

$$R_{23}=\left\{16\sqrt{\frac{E_{11}}{E_{22}}}+72\nu_{21}\psi^2\sqrt{\frac{E_{11}}{E_{22}}}+81\psi^4\sqrt{\frac{E_{22}}{E_{11}}}+144\psi^2\frac{G_{12}(1-\nu_{12}\nu_{21})}{\sqrt{E_{11}E_{22}}}-4\psi^2(1-\frac{c}{2})k_b\right\}$$

$$R_{31}=\left\{81\sqrt{\frac{E_{11}}{E_{22}}}+18\nu_{21}\psi^2\sqrt{\frac{E_{11}}{E_{22}}}+\psi^4\sqrt{\frac{E_{22}}{E_{11}}}+36\psi^2\frac{G_{12}(1-\nu_{12}\nu_{21})}{\sqrt{E_{11}E_{22}}}-9\psi^2(1-\frac{c}{2})k_b\right\}$$

$$R_{32}=\left\{81\sqrt{\frac{E_{11}}{E_{22}}}+72\nu_{21}\psi^2\sqrt{\frac{E_{11}}{E_{22}}}+16\psi^4\sqrt{\frac{E_{22}}{E_{11}}}+144\psi^2\frac{G_{12}(1-\nu_{12}\nu_{21})}{\sqrt{E_{11}E_{22}}}-9\psi^2(1-\frac{c}{2})k_b\right\}$$

$$R_{33}=\left\{81\sqrt{\frac{E_{11}}{E_{22}}}+162\nu_{21}\psi^2\sqrt{\frac{E_{11}}{E_{22}}}+81\psi^4\sqrt{\frac{E_{22}}{E_{11}}}+324\psi^2\frac{G_{12}(1-\nu_{12}\nu_{21})}{\sqrt{E_{11}E_{22}}}-9\psi^2(1-\frac{c}{2})k_b\right\}$$

$$R_{41}=\left\{256\sqrt{\frac{E_{11}}{E_{22}}}+32\nu_{21}\psi^2\sqrt{\frac{E_{11}}{E_{22}}}+\psi^4\sqrt{\frac{E_{22}}{E_{11}}}+64\psi^2\frac{G_{12}(1-\nu_{12}\nu_{21})}{\sqrt{E_{11}E_{22}}}-16\psi^2(1-\frac{c}{2})k_b\right\}$$

$$k_\beta=\frac{16c\psi^2 k_b}{\pi^2}$$

Unit Conversion:

1 ksi $=$ 1000 lb/in^2
1 lb $=$ 0.4536 kg
1 in $=$ 2.54 cm