

論文

The Influence of Transverse Shear Stress on Damping of 0-degree Unidirectional Laminated Composites

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0-degree 단일 방향 적층된 복합재료의 댐핑에 대한 횡전단 응력의 영향

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초 록

적층된 복합재료의 댐핑(Damping)을 예견하는 한 모델이 스트레인 에너지 소산량의 개념을 근거로 개발되어져왔다. 본 모델속에 댐핑의 횡전단 응력 효과가 평면 신장/압축과 평면 전단응력위에 포함되어져왔다. 그 모델의 유효성이 0 도 일축방향 복합재료 보의 다양한 길이와 두께를 변화시켜서 댐핑을 측정함으로써 수행되어졌다.

0 도 적층된 복합재료 댐핑의 이론적 예측의 결과들은 실험적 측정값들과 호의적으로 비교되어지는 것으로 발견되었다. 그 횡전단 응력은 일축방향 고분자 복합재료의 댐핑에 커다란 영향을 미치는 것으로 나타났다.

ABSTRACT

A model for predicting damping in composites has been developed based on the concept of strain energy-weighted dissipation. In this model, the effect of transverse shear stress on damping has been included in addition to the effects of inplane extension/compression and inplane shear. Validation of the model was attempted by conducting damping measurements on 00 unidirectional composite beams with varying length and thickness. The results of theoretical predictions of damping in laminated composites were found to compare favorably with experimental data. The transverse shear appears to have a significant effect on the damping mechanisms in 0° unidirectional polymer composites.

I . INTRODUCTION

Damping is an important factor in the design of the structure with controlled vibration and movement. In many structural applications, damping provides sufficient energy dissipation to suppress resonant amplitudes of vibration. In general, damping arises from mul-

tiple sources such as the inherent material damping [1], aerodynamic damping, and structural damping. In structures, additional sources of damping are provided through energy dissipation mechanisms such as bolted, riveted and bonded joints [1]. From a design perspective, material damping can be viewed as the reliable lower bound on energy dis-

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sipation. The degree of structural damping is sensitive to fabrication and assembly details that may vary.

Several researchers have developed the transverse shear models on damping in laminated composites. Bicos and Springer[2] explored a dynamic response of shell structure by using the damped element model through a finite element method(FEM). Alam and Asnani[3] employed the concept of the correspondence principle of linear viscoelasticity and the solution in a series form for damping analysis of rectangular plate. Extension, bending, in-plane shear and transverse shear deformation were included in their damping model. Saravanos et. al.[4] investigated the integrated damping mechanics for composite plates with constrained interlaminar layers of polymer damping materials. Their works involve high-order or discrete layer theories and address more complex structural members. Hwang and Gibson[5] developed a 3-D FEM to take into account the contribution of the interlaminar stresses. The interlaminar damping was found to be particularly important over a particular range of ply orientation. Koo and Lee[6] studied the damping of composite laminates using FEM on the basis of the shear deformable plate theory. But they did not provide the experimental data.

In this paper, we developed a straightforward and simple model taking account of the transverse shear stress to predict the damping of laminated composites based on Adams and Ni's theory.

Material damping can be defined as any process that transforms the energy of a mechanical vibration into some other form of irrecoverable energy. From an energy viewpoint, material damping is defined as the ratio of the change in stored energy (dissipated energy), ΔW , to the maximum stored energy, W , during a cycle as shown in Fig.(1). Throughout this paper, material damping is represented in

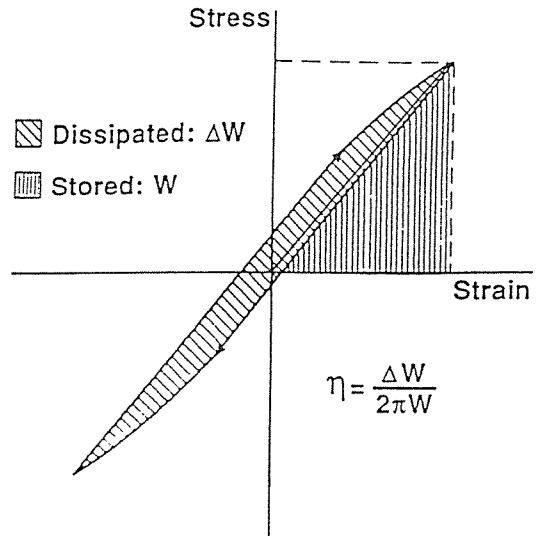


Fig. 1. Energy dissipation during a loading cycle.

terms of the damping loss factor, η :

$$\eta = \Delta W / (2\pi W)$$

In composites, damping is normally attributed to one or more of the following mechanisms [1,7]: (1) Viscoelastic response of the constituents; (2) Thermoelastic damping due to cyclic heat flow; (3) Friction at the fiber/matrix interface; (4) Damage initiation and growth.

Material damping represents the cumulative contributions of the viscoelastic response of the constituents, cyclic heat flow and the friction at the fiber/matrix interface. Materials with internal interfaces, as exemplified by the interfaces of the laminae of conventional composite materials, can dissipate mechanical energy through the movement of the interface by mechanisms such as a discontinuity of stress across the interface or by frictional losses. Und-

er vibration, a considerable amount of heat is generated in composite materials, which is indicative of high energy losses in the material. Nondestructive evaluation techniques such as IR Thermography [12] have recently demonstrated this energy loss. The heat generated due to material imperfections is caused by friction at defects, such as cracks in fibers and the matrix, and at delaminations.

Although it is not possible to isolate the individual mechanisms for a specific material, from the viewpoint of mechanics, a damping prediction model can be analytically developed by considering inplane stresses as well as the transverse shear stress. By adding transverse shear deformation, we can quantify the sensitivity of damping loss factor with respect to the variation in thickness and length of laminated beams. The numerical results based on such a model have been validated with experimental data.

II. THEORETICAL BACKGROUND

Laminated beam formulae have been developed to account for the stacking sequence of individual plies based upon Tsai's [13] and Adams and Ni's [14] works. The formulation is built on the coordinate system as shown in Fig. (2). In this research, a principal flexural moment, M_1 , is applied to the laminated beam. Due to this flexural moment, physically transverse shear stresses in the beam exist. Therefore, the equations of motion must account for the interlaminar stresses. These out-of-plane stresses in the beam can easily be determined by assuming that these stresses are independent of the y -axis because the strip of beam is oriented along the x -axis. Thus, the energy dissipated can be separated into 4 different sources, being associated with the longitudinal (σ_x), transverse (σ_y), in-plane shear (σ_{xy}), and transverse shear (σ_{xz}) stresses, respectively, in fiber coordinates. The e-

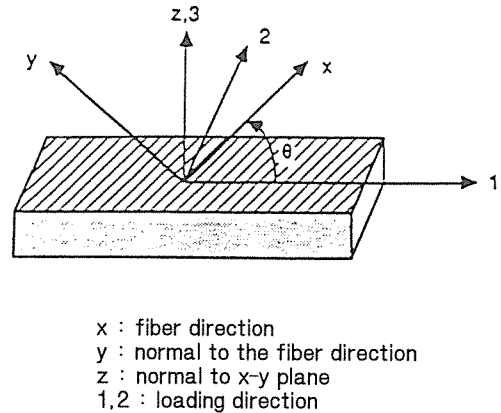


Fig. 2. Fiber and loading coordinate systems for theoretical model.

quilibrium equations for the t -th layer of the laminate are described as follows :

$$\frac{\partial \sigma_x^{(t)}}{\partial x} + \frac{\partial \sigma_y^{(t)}}{\partial y} + \frac{\partial \sigma_z^{(t)}}{\partial z} = 0 \quad \dots\dots\dots (1)$$

$$\frac{\partial \sigma_y^{(t)}}{\partial x} + \frac{\partial \sigma_x^{(t)}}{\partial y} + \frac{\partial \sigma_z^{(t)}}{\partial z} = 0 \quad \dots\dots\dots (2)$$

$$\frac{\partial \sigma_x^{(t)}}{\partial x} + \frac{\partial \sigma_y^{(t)}}{\partial y} + \frac{\partial \sigma_z^{(t)}}{\partial z} + F_z = 0 \quad \dots\dots\dots (3)$$

Hence, in the energy approach both flexural strain energy and transverse shear strain energy are taken into account. To analyze the transverse shear deformation effect on damping, the flexural bending moment must be assumed to fit experimental conditions. Two loading conditions were considered in the present effort to determine the suitable flexural bending moment. One is point loading with $M_1 = Px$ while the other is distributed mass loading with $M_1 = 1/2 wx^2$. Finally, the energy dissipation for any 0° laminate can be predicted by using the energy method and the beam theory.

2-1. FLEXURAL BENDING MOMENT

According to Adams and Ni's [14], the strain energy dissipation which is subjected to bending in the beam can be divided into three parts related to inplane stresses such as s_x , s_y and s_{xy} in the fiber coordinate.

Simply,

$$\Delta W = \Delta W_x + \Delta W_y + \Delta W_{xy} \quad (4)$$

The strain energy dissipation about ox is written as

$$\begin{aligned} \Delta W_x &= \int_0^l 2 \int_0^{h/2} \pi \eta_L \sigma_x \epsilon_x dz dx = 2\pi \eta_L \int_0^l \int_0^{h/2} \sigma_x \epsilon_x dz dx \\ &= \frac{2\pi \eta_L}{I^{*2}} \int_0^{h/2} m^2 (Q_{11} d_{11}^* + Q_{12} d_{12}^* + Q_{16} d_{16}^*) \\ &\quad (m^2 d_{11}^* + mnd_{16}^*) z^2 dz \int_0^l M_1^2 dx \quad (5) \end{aligned}$$

where,

l = length of beam, h = thickness of beam,
 η_L = axial basic damping loss factor, d_{ij}^* = normalized flexural compliance, $M_1 = Px$ = bending moment under point loading or,
 $M_1 = 1/2 wx^2$ = bending moment under distributed loading.

Similarly, ΔW_y and ΔW_{xy} can be evaluated as follows;

$$\begin{aligned} \Delta W_y &= \frac{2\pi \eta_T}{I^{*2}} \int_0^{h/2} n^2 (Q_{11} d_{11}^* + Q_{12} d_{12}^* + Q_{16} d_{16}^*) \\ &\quad (n^2 d_{11}^* - mnd_{16}^*) z^2 dz \int_0^l M_1^2 dx \quad (6) \\ \Delta W_{xy} &= \frac{2\pi \eta_{LT}}{I^{*2}} \int_0^{h/2} mn (Q_{11} d_{11}^* + Q_{12} d_{12}^* + Q_{16} d_{16}^*) \\ &\quad (2mnd_{11}^* - (m^2 - n^2) d_{16}^*) z^2 dz \int_0^l M_1^2 dx \quad (7) \end{aligned}$$

where, η_T = transverse basic damping loss factor, η_{LT} = in-plane shear basic damping loss factor, I^* = the normalizing factor which equals $h^3/12$.

And the bending strain energy of the beam is

$$W_b = \int_0^l M_1 k_1 dx = \frac{d_{11}^*}{I^*} \int_0^l M_1^2 dx \quad (8)$$

2-2. TRANSVERSE SHEAR STRESS

From the equilibrium equations (1)-(3) of the laminate, the transverse shear stress can be obtained. In order to apply the equilibrium equation, we first substituted the following in-plane stresses into the equilibrium equation.

$$\begin{aligned} \sigma_x^{(i)} &= \frac{zM_1}{I^*} m^2 (Q_{11}^{(i)} d_{11}^* + Q_{12}^{(i)} d_{12}^* + Q_{16}^{(i)} d_{16}^*) \\ &= \frac{zf_1^{(i)} M_1}{I^*} \quad (9) \end{aligned}$$

$$\begin{aligned} \sigma_y^{(i)} &= \frac{zM_1}{I^*} n^2 (Q_{11}^{(i)} d_{11}^* + Q_{12}^{(i)} d_{12}^* + Q_{16}^{(i)} d_{16}^*) \\ &= \frac{zf_2^{(i)} M_1}{I^*} \quad (10) \end{aligned}$$

$$\begin{aligned} \sigma_{xy}^{(i)} &= \frac{zM_1}{I^*} (-mn) (Q_{11}^{(i)} d_{11}^* + Q_{12}^{(i)} d_{12}^* + Q_{16}^{(i)} d_{16}^*) \\ &= \frac{zf_3^{(i)} M_1}{I^*} \quad (11) \end{aligned}$$

where,

$$f_1^{(i)} = m^2 (Q_{11}^{(i)} d_{11}^* + Q_{12}^{(i)} d_{12}^* + Q_{16}^{(i)} d_{16}^*) \quad (12)$$

$$f_2^{(i)} = n^2 (Q_{11}^{(i)} d_{11}^* + Q_{12}^{(i)} d_{12}^* + Q_{16}^{(i)} d_{16}^*) \quad (13)$$

$$f_3^{(i)} = (-mn) (Q_{11}^{(i)} d_{11}^* + Q_{12}^{(i)} d_{12}^* + Q_{16}^{(i)} d_{16}^*) \quad (14)$$

For laminated composite beams, the interlaminar shear stress, σ_{xz} , is determined from the equilibrium equation (1) by assuming the axis of the beam to run along x-axis. The stresses can be assumed to be independent of the y-axis because the strip of beam is oriented along the xaxis. Under these assumptions, we can obtain the interlaminar shear stress (σ_{xz}). However, this model is valid only for 0° unidirectional composites for two reasons. The first reason is that the loading direction coin-

cides with fiber direction only in unidirectional composites. The second reason is that we could not explicitly quantify interlaminar shear modulus with variations of each fiber angle, except for transversely isotropic unidirectional composites. Therefore, we can take the transverse shear deformation effect into account only in the case of 0° unidirectional composites. By applying the above equations we have:

$$\sigma_{xz}^{(i)} = -\frac{1}{I^*} \int_{-h/2}^{z_i} f_1^{(i)} \frac{dM_1}{dx} dz \quad \dots\dots\dots (15)$$

From the beam theory, the $\sigma_{xz}^{(i)}$ term was obtained as follows:

$$\sigma_{xz}^{(i)} = -\frac{Q}{I^*} \int_{-h/2}^{z_i} f_1^{(i)} dz \quad \dots\dots\dots (16)$$

where, $Q = P =$ bending force in the point loading, or, $Q = wx =$ bending force in the distributed loading.

The integral Eq.(16) assures continuity of the transverse shear stresses at layer interfaces. The shear strain energy of the beam is written as

$$W_{shear} = \int_{vol} \frac{\sigma_{xz}^2}{2G_c} dv = \frac{1}{2G_c} \int_0^l \int_{-h/2}^{h/2} \sigma_{xz}^2 b dz dx \quad \dots\dots (17)$$

where, $G_c = G_m \frac{G_f}{(1 - V_f)G_f + V_f G_m}$, $G_c =$ shear modulus of lamina and $G_f =$ shear modulus of fiber, $G_m =$ shear modulus of matrix, $V_f =$ fiber volume fraction.

The dissipated energy that is related to interlaminar stress, σ_{xz} , is written as

$$\Delta W_{xz} = \frac{\pi \eta_s}{G_c} \int_0^l \int_{-h/2}^{h/2} \sigma_{xz}^2 b dz dx \quad \dots\dots\dots (18)$$

where, $\eta_s = \frac{(1 - V_f)\eta_m G_c}{G_m} =$ transverse basic

shear damping loss factor [11] in the case of 0° unidirectional composites, and $\eta_s = \eta_{LT}$, $\eta_m =$ matrix damping loss factor, $G_m =$ matrix shear modulus, $G_c =$ composite shear modulus.

2-3. TOTAL STORED ENERGY IN THE CANTILEVER BEAM

The total stored energy in the beam is described as

$$W = W_b + W_s$$

$$= \frac{d_{11}^*}{I^*} \int_0^l M_1^* dx + \frac{1}{2G_c} \int_0^l \int_{-h/2}^{h/2} \sigma_{xz}^2 b dz dx \quad \dots\dots\dots (19)$$

where, $W_b =$ bending strain energy in the cantilever beam and $W_s =$ transverse shear strain energy in the cantilever beam.

From results of the above energy method, the total damping loss factor is predicted as:

$$\psi_{ov} = \frac{\sum \Delta W}{\sum W} = \frac{\Delta W_x + \Delta W_y + \Delta W_{xy} + \Delta W_{xz}}{W_b + W_s} \quad \dots\dots (20)$$

where, $\psi_{ov} = 2\pi\eta_{ov} =$ specific damping capacity and $\eta_{ov} =$ overall damping loss factor.

2-4. DETERMINATION OF BASIC DAMPING LOSS FACTORS

In order to calculate the dissipated energy in composites, it is essential to accurately evaluate the basic damping loss factors. In general, it is known that an increase in the amount of damage in the material, the stress amplitude of the test, or the test frequency tends to enhance the damping loss factor.

The theoretical models that are currently available to predict the damping loss factor for composites are inadequate for design purposes. A theoretical model [15] assumes that adhesives are flexible and the bonding between the fiber and the resin is perfect. However, practically, it is very difficult to fa-

bricate specimens that satisfy these assumptions. Also, damping can be caused by interface mechanisms such as shearing motion between the fiber and the matrix. In addition, although fiber damping is usually assumed to be negligible, it is necessary to exactly be elucidated in the future works. Further, as the structural dimensions increase, the number of defects in the material increase as well. Accordingly, the basic damping loss factors were statistically determined by experimental works using numerous unidirectional 0° , 45° , 90° (fiber direction) specimens.

III. EXPERIMENTAL

In this study, the impulse technique was used for measuring the material damping. Major components of the apparatus include the test specimen, a clamping block, an electromagnetic hammer, and a noncontact eddy current probe. The test specimen is a flat laminate that is supported as a cantilever beam in the clamping block. All test specimens had a width of one inch and a thickness of either 8(0.042 inch), 20(0.1 inch) or 32(0.162 inch) plies. The specimen length was chosen to provide the desired frequency to facilitate the measurement of the loss factor. The electromagnetic hammer was used to provide a reproducible impulse. The noncontact eddy current probe is a motion transducer located at the tip of the specimen. The signals from the force and motion transducer are input into a Fast Fourier Transform (FFT) analyzer. As a method of data processing for loss factors the half-power bandwidth method of frequency resolution in zoom mode was applied to the frequency response function curve to evaluate the loss factor. A schematic drawing of the experimental setup is presented in Fig.(3). A detailed description of this experimental method can be found in ref.[7, 8].

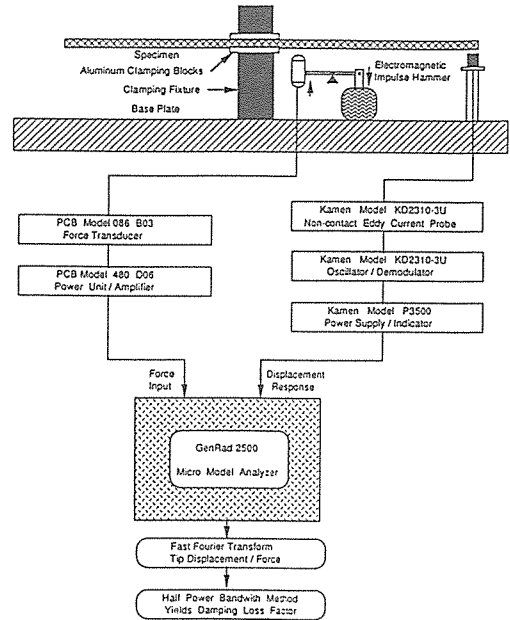


Fig. 3. Instrumentation for impulse Technique.

IV. RESULTS AND DISCUSSION

The variations of the damping loss factor with respect to thickness and length are shown in Fig.(4)-(7). Experimental data for a 20ply($t=0.1$ inch, $w=1$ inch) AS4/3501-6 composite are plotted in Fig.(5) along with the theoretical curve. Although the predicted damping data are lower than the experimental data, there is a reasonable agreement between them. In the cantilever beam apparatus, there might still be some damping in the grips even though the grips were tight [16]. In this study there exists more extraneous loss in the measured damping of a short beam because of a grip effect. For the 0° AS4/3501-6 composites, the damping loss factor increases with decreasing beam length.

Damping of the thicker beam is more significantly influenced than that of the thinner beam. The influence of specimen thickness on

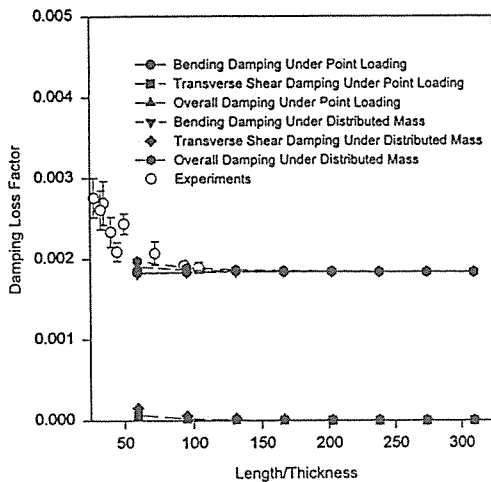


Fig. 4. Theoretically and experimentally determined vibration damping loss factor versus specimen length/thickness for 0 degree 8 ply AS4-3501-6 composites.

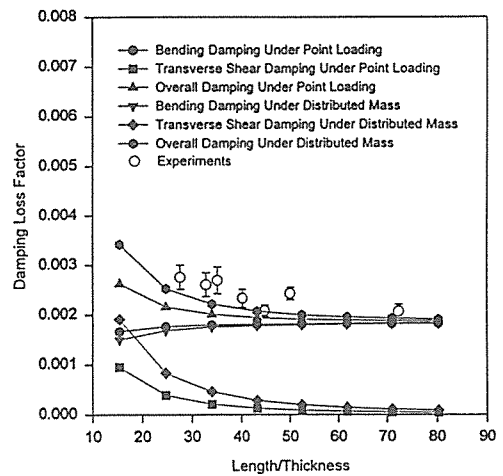


Fig. 6. Theoretically and experimentally determined vibration damping loss factor versus specimen length/thickness for 0 degree 32 ply AS4/35016 composites.

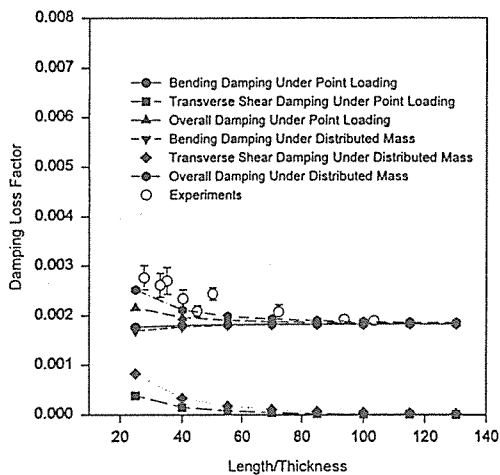


Fig. 5. Theoretically and experimentally determined vibration damping loss factor versus specimen length/thickness for 0 degree 20 ply AS4/35016 composites.

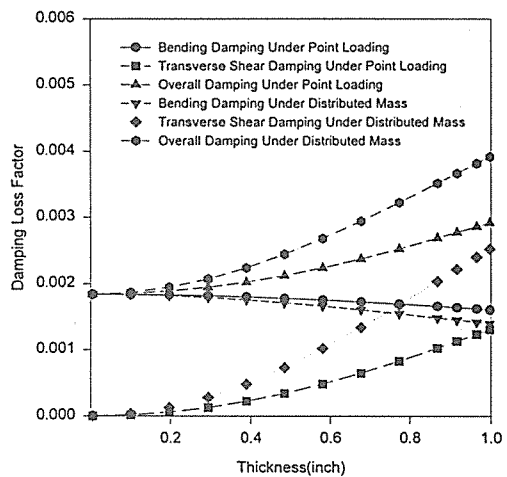


Fig. 7. Theoretically determined vibration damping loss factor versus specimen thickness for 0 degree AS4/35016 composites on 13 inch specimen length.

the damping loss factor of AS4/3501-6 composites was examined by using 8 (0.042 inch), 20 (0.1 inch) and 32 ply (0.162 inch) lam-

inates. In this study, even though the loss factor shows a gradual increase with increasing thickness of the specimen analytically, there

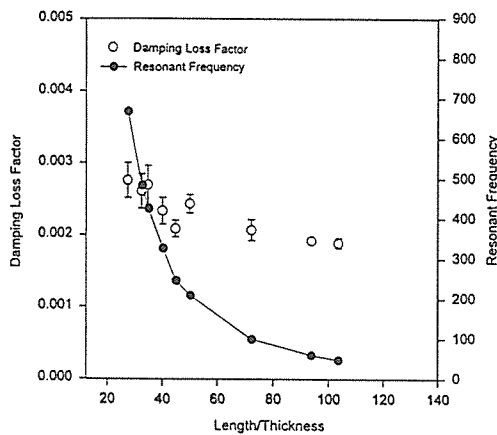


Fig. 8. Damping loss factor and frequency versus specimen length/thickness for 0 degree 20 ply AS4/3501-6 composites.

Table 1. Basic Material Properties of AS4/3501-6 Composites Used in This Study

E_L (GPa)	E_T (GPa)	ν_{LT}	G_{LT} (GPa)	η_L ($\times 10^{-3}$)	η_T ($\times 10^{-3}$)	η_{LT} ($\times 10^{-3}$)
128	9.26	0.3	5.9	1.8405	8.5801	9.477

is no clear distinction in the loss factors among them (8ply, 20ply and 32ply) due to excessive experimental data scattering. The damping loss factor appears to increase with increasing thickness of the specimen owing to the effect of transverse shear (Fig.(7)). Fig. (8) shows the loss factors and frequencies versus specimen length/thickness in mode 1 shape. The determination of an optimized length and thickness for a structure of which the structural damping is at a maximum is very important for designing a structure with good dynamic performance. As a result of two loading cases for the suitable flexural moment, the damping values obtained by the distributed mass loading were closer to experimental damping data than those by the point loading.

Table 2. AS4/3501-6 [0]₂₀ Graphite/Epoxy Damping Data

V. CONCLUSIONS

A general mathematical model has been developed for predicting the damping of laminated composite beams based upon the work of Adams and Ni[14]. This model is more comprehensive than that of Adams and Ni and results in a significant improvement in the prediction of damping on short and thick 0° unidirectional composite beams even though the experimental results departed from the theoretical predictions as the beam became short. Reasonable agreement was found between theoretical values and experimental data. Material damping increases with increasing thickness and decreasing length of the specimen due to transverse shear stress (σ_{13}). Thicker specimens are more sensitive than thinner ones to material damping.

ACKNOWLEDGMENTS

Support for this project was provided by the NSF/Alabama EPSCoR program to which we are grateful.

REFERENCES

1. Adams, R.D., Damping Properties Analysis of Composites, Engineering Materials Handbook, Composites, Vol. 1, pp.206-217, ASM(1987).
2. Bicos, A.S. and Springer, G.S., "Analysis of Free Damped Vibration of Laminated Composite Plates and Shells," International Journal of Solids and Structures, Vol. 25, pp.129-149, 1989.
3. Alam, N. and Asnani, N.T., "Vibration and Damping Analysis of Fibre Reinforced Composite Material Plates," Journal of Composite Materials, Vol. 20, pp.2-18, 1986.
4. Saravanas, D.A., "Integrated Damping

Table 2. AS4/3501-6 [0]₂₀ Graphite/Epoxy Damping Data

Beam length(inch)		First Mode		Beam length(inch)		First Mode	
		F ₁ (Hz)	Loss factor(x10 ⁻⁴)			F ₁ (Hz)	Loss factor(x10 ⁻⁴)
10.35	1	48.54	18.2	4.02	1	326.04	21.3
	2	48.55	19.6		2	326.32	23.8
	3	48.54	18.9		3	326.54	24.9
	Avg.	48.543	18.9		Avg.	326.30	23.3
	S.D.	0.00577	0.7		S.D.	0.251	1.84
9.37	1	60.42	19.6	3.50	1	424.84	29.1
	2	60.43	19.2		2	425.48	23.9
	3	60.54	18.8		3	425.80	27.8
	Avg.	60.463	19.2		Avg.	425.373	26.9
	S.D.	0.0666	0.4		S.D.	0.488	2.71
7.24	1	100.27	20.8	3.27	1	484.81	25.1
	2	100.32	19.2		2	484.90	24.3
	3	100.62	22.1		3	484.99	28.8
	Avg.	100.403	20.7		Avg.	484.90	26.06
	S.D.	0.189	1.45		S.D.	0.090	2.40
5.00	1	209.19	23.2	2.76	1	668.44	2.40
	2	209.19	23.2		2	668.77	29.4
	3	209.42	24.1		3	668.82	24.8
	Avg.	209.33	24.3		Avg.	668.677	27.6
	S.D.	0.123	1.26		S.D.	0.206	2.44
4.48	1	246.42	20.3				
	2	246.20	20.3				
	3	246.51	20.1				
	Avg.	246.376	20.8				
	S.D.	0.159	1.16				

Mechanics for Thick Composite Laminates and Plates," Journal of Applied Mechanics, Vol. 61, pp.375-383,1994.

5. Hwang, S.J. and Gibson, R.F., "Contribution of Interlaminar Technology, 41, pp. 379-393,1991.

6. Koo, K.N. and Lee, I., "Vibration and Damping Analysis of Composite Laminates Using Shear Deformable Finite Element, "AIAA J., Vol. 31, No.4, April, pp.728-735, 1993.

7. Yim, J.H., Burmeister, J.S., Kaminski, R. L. and Gillespie, J.W. Jr., Experimental Characterization of Material Damping in Laminated Polymer Matrix Composites, Center of

Composite Materials Report 1988-39, University of Delaware, Newark, Delaware.

8. Crane, R.M. and Gillespie, J.W. Jr., "Characterization of the Vibration Damping Loss Factor of Glass and Graphite Fiber Composites,"Composites Science and Technology Vol. 40, pp.355-375,1991.

9. Spirnak, G.T. and Vinson, J.R., "The Effect of Temperature on the Material Damping of Graphite/Epoxy Composites in a Simulated Space Environment," Recent Advances in the Micro- and Macro-Mechanics of Composite Material Structures, Editors: D. Hui and Vinson, ASME Publication AD-Vol. 13, pp.189-192,

1988.

10. Gibson, R.F. and Plunkett, R., "Dynamic Mechanical Behavior of Fiber Reinforced Composites: Measurement and Analysis," Journal of Composite Materials, Vol. 10, pp. 325-341, 1976.

11. Willway, T.A. and White, R.G., "The Effects of Matrix Complex Moduli on the Dynamic Properties of CFRP Laminate," Composites Science and Technology, Vol. 36, pp.77-94, 1989.

12. Henneke, E. G., II and Jones, T. S., "Detection of Damage in Composite Materials by Vibrothermography," Nondestructive Evaluation and Flaw Criticality of Composite Materials, ASTM STP 696, R. B. Pipes, Ed., American Society for Testing and Materials, pp.83-95, 1979.

13. Tsai, S.W., "Composites Design," Fourth Edition, Think Composites, Dayton, Ohio, 1988.

14. Ni, R.G. and Adams, R.D., "The Damping and Dynamic Moduli of Symmetric Laminated Composite Beams- Theoretical and Experimental Results," J. Comp. Matls, Vol. 18, p.104(1984).

15. Hashin, Z., "Complex Moduli of Viscoelastic Composites: II. Fiber Reinforced Materials," Int. J. Solids and Struc., Vol. 6 pp.797-807 (1970).

16. Wren, G.G. and Kinra, V.K., "An Experimental Study of the Complex Dynamic Modulus," Dynamic Elastic Modulus Measurements in Materials, ASTM STP 1045, pp. 58-74.
